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Diagnosis of the arithmetic problems understanding in the primary education

El diagnóstico de la comprensión de problemas aritméticos en la educación primaria

O diagnóstico da compreensão dos problemas aritméticos no ensino primário

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ABSTRACT
The understanding constitutes an essential element in the teaching-learning process of the arithmetic solving problems. Nevertheless, in it, it has predominated the conception that lays stress on the result that limits the precision of achievements and student's insufficiencies, his possible causes and the instrumentation of an adequate intervention. Logically, held in a conception that the solution of problems like a process of textual understanding, in the article defends, they offer theoretical and methodology considerations on his diagnosis. In the investigation, various empiric and Theoretic
methods, Such as documentary analysis, synthesis analysis, induction and deduction and modelation were used. Among the main results that they make are: the re contextualization of dimensions, indicators and levels of performance of the textual understanding to the words problems solving.

Key words: arithmetic problems; diagn ostic; Mathematics Education; pedagogical test; understanding.

RESUMEN
La comprensión constituye un elemento esencial en el proceso de enseñanza-aprendizaje de la solución de problemas aritméticos. No obstante, en este último ha predominado la concepción que pone énfasis en el resultado, lo que limita la precisión de los logros e insuficiencias de los escolares, sus posibles causas y la instrumentación de una adecuada intervención. Consecuentemente, sustentados en una concepción que defiende la solución de problemas como un proceso de comprensión textual, en el artículo se ofrecen consideraciones teórico-metodológicas sobre su diagnóstico. En la investigación se emplearon diversos métodos empíricos y teóricos, tales como: el análisis documental, el análisis-síntesis, la inducción-deducción y la modelación. Entre los principales resultados que se aportan están: la recontextualización de dimensiones, indicadores y niveles de desempeño de la comprensión textual a la solución de problemas aritméticos verbales.

Palabras clave: comprensión; diagnóstico; Didáctica de la Matemática; problema aritmético; prueba pedagógica.

RESUMO
A compreensão é um elemento essencial no processo ensino-aprendizagem para a resolução de problemas aritméticos. No entanto, neste último, predominou o conceito que enfatiza os resultados, o que limita a precisão das realizações e deficiências dos estudantes, as suas possíveis causas, e a implementação de uma intervenção adequada. Consequentemente, com base numa concepção que defende a solução dos problemas como um processo de compreensão textual, o artigo oferece considerações teórico-metodológicas sobre o seu diagnóstico. Na investigação, foram utilizados vários métodos empíricos e teóricos, tais como: análise documental, análise-síntese, indução-dedução e modelação. Entre os principais resultados fornecidos estão: a recontextualização de dimensões, indicadores e níveis de desempenho desde a compreensão textual até à solução de problemas de aritmética verbal.

Palavras-chave: Compreensão; Diagnóstico; Didática da Matemática; Problema aritmético; Teste pedagógico.

INTRODUCTION

The study of the consulted sources: Blanco & Caballero (2015); Pérez, Álvarez & Breña (2016); Pérez & Hernández (2020), allowed to identify various characterizations of the concept of arithmetic problem. Its subordination to the problem, justifies that the author of the article assumes the criterion of Campistrous & Rizo (1996), when they state that a problem is: "(…) any situation in which there is an initial approach and a requirement that forces to transform it" (p. 9). They also add two conditions: the solution path is unknown by the solver and the latter wishes to find it (Campistrous & Rizo, 1996).

The interest in the approach to arithmetic problems, which are formulated verbally, conditions the need to further reduce the subordinate concept of arithmetic problem. Hence, the author of the article, assumes - for its subdivision - the criterion that meets the code used (Pérez & Hernández, 2020), from which the Hispanic and English-speaking nomenclatures emerge, respectively, of "verbal arithmetic problem" or "of verbal statement" (PAEV) and "word problems".

Consequently, when the term arithmetic problem is used, the article will be referring to those problems that, in addition to complying with the demands set forth by Campistrous & Rizo (1996), are formulated verbally and to solve them the use of At least - one arithmetic calculation operation.

Following the line of thought of Pérez *et al.* (2016); Pérez (2017); Pérez, Coaguila & Hernández (2019), those who defend the textual nature of arithmetic problems, assume the conception that explains their solution as a process of textual comprehension, defended by authors such as: Pérez & Hernández (2020) and Montero & Mahecha (2020). According to Pérez & Hernández (2020), textual comprehension implies cognitive and affective processes; Therefore, in the article, the criteria of Pérez & Hernández (2015) are assumed, who define the understanding of an arithmetic problem as: "(…) the activity aimed at the search for the relationships contained in a text necessary to satisfy the requirement (s) of the problem and make a comprehensive assessment of the text"(p. 21).

From the position assumed, the importance of the cognitive performance category is recognized, which is closely related to that of levels of cognitive performance, defined by Cruz, Romero & Marrero (2020) as: "(…) the degree of complexity with which we want to measure this cognitive performance and, at the same time, the

magnitude of the learning achievements achieved in a given subject" (p. 281). For the instrumentation of its categorizing function, the position of Cruz *et al.* (2020) is assumed, regarding the need to use the percentage analysis of correct answers by performance levels.

The study of the contributions of Jiménez (2005a and b); Leyva, Proenza, Leyva, Cristo & Romero (2008) and Hernández (2012), reveal a certain reduction in the instrumentation of the levels of cognitive performance of comprehension, when treating the subject Spanish Language. In addition, the consideration of the ontogenetic development of primary schoolchildren is considered limited for the precision of performance levels and their corresponding cognitive operations in the process of textual comprehension.

The position assumed implies, in turn, the re-dimensioning of the direction of their teaching- learning, in which diagnosis is a core element. In the words of Perez (2020), in the specialized literature can be identified, that despite the existence of various proposals for operationalization and instruments for their implementation, are influenced by the conception that assumes understanding as previous step in arithmetic problems.

In addition, Perez & Hernandez (2017) consider that the existing proposals put greater emphasis on the result that in the process of understanding during the problems solution, influenced significantly by studies assessing the quality of learning, in which it is assumed as the essence of the higher level of cognitive performance in any cognitive domain and area of knowledge. Following that line of thought, in the article are dimensions, indicators and performance cognitive levels of textual understanding to solving arithmetic problems are re contextualized.

MATERIALS AND METHODS

The use of the analytic-synthetic and inductive-deductive methods served to assume arithmetic problems as text and their solution as a process of textual comprehension. On the other hand, the review of the specialized literature allowed identifying that the proposed instrumentations, for the diagnosis of the understanding of arithmetic problems, r espond to the conception that assumes to the first as a previous stage of the second. Modeling was used to re contextualize dimensions, indicators and levels of cognitive performance from text comprehension to solving arithmetic problems.

RESULTS

Dimensions, indicators and performance levels to diagnose understanding of arithmetic problems

From the postulates of the developer Didactics, assumed in the work, the diagnosis constitutes a starting point (principle) of the direction of the teaching-learning process; in addition, its procedural nature is recognized. In line with this, it is defended as an essential element for the structuring of the didactic treatment of the understanding of verbal arithmetic problems in primary education.

The diagnosis of understanding verbal arithmetic problems defined in this article as the process of determining the potential and limitations of the school to understand verbal arithmetic problems. Its objective is aimed at obtaining information on the cognitive performance of schoolchildren, in terms of understanding in the solution of verbal arithmetic problems.

Starting from the assumption of understanding as a process of production of meanings and by conceiving the logical-subject matter and the socio-referential as its basic axes of meaning, it is necessary to re signify its operationalization. In keeping with this, the following dimensions and indicators are conceived:

Dimension 1. Cognitive of the text

Indicator 1. Generation: consists on the elaboration of ideas through inferences, generalizations, etc.

Indicator 2. Transformation: it lies in the deduction of implicit meanings and in the translation of textual elements from one code to another or from one system of signs to others.

Indicator 3. Flexibility: it means the possibility of managing several alternatives of meaning (relationships).

Taking into account the particularities of understanding in solving arithmetic problems, the first three indicators are measured in an integrated way and it is called flexible generation. For this, the following ordinal scale is used:

- Very Low (MB): It only identifies explicit information.

- Low (B): It identifies implicit information from making simple inferences.

- Medium (M): It elaborates inferences works from temporal relationships, information of part completely found or other types; furthermore, it partially translates into mathematical code.

- High (A): and makes inferences integrating various relationships It elaborates (partiality), makes evaluative judgments about the socio-referential information of the problem and translates coherently to the mathematical code.

- Very High (MA): creates different meanings from integrating several complex relationships and translates consistently to the mathematical code.

Indicator 4. Autonomy: implies independence in the execution of the process. It is valued, according to the following scale:

- Very Low (MB): executes the activity in an incomplete, slow and dependent way.

- Low (B): Executes the activity in a way: incomplete, fast and dependent or incomplete, fast and independent.

- Medium (M): Executes the activity in a way: complete, slow and dependent or incomplete, fast and dependent.

- High (A): Executes the activity in a way: complete, slow and independent or complete, fast and dependent.

- Very High (MA): Executes the activity completely, quickly and independently.

Dimension 2. Valuation of the text

Indicator 5. Selection of values: it resides in the discovery of the essential values that are derived from the content of verbal arithmetic problems.

Indicator 6. Critical position: consists of the formation of evaluative judgments based on the semantic content of verbal arithmetic problems.

The indicators of this dimension are measured in an integrated way, being called: evaluation criteria. The ordinal scale used for this is as follows:

- Very Low (MB): It gives imprecise and insufficient criteria.

- Low (B): It offers partially founded criteria with little coherence.

- Medium (M): It offers partially grounded criteria with consistency.

- High (A): It offers founded and consistent criteria in which they are clear features of personal development and emotional attachment.

- Very High (MA): It offers well-founded and coherent criteria in which a high degree of personal elaboration and emotional bond is appreciated.

Given the link of the aforementioned dimensions with the cognitive and affective processes, it is required - to achieve a comprehensive diagnosis of schoolchildren - the use of various methods, techniques and instruments that allow a multilateral assessment of the process under analysis. It goes beyond the current and predominant conception - for several decades - of emphasizing the cognitive (conceptual and procedural); and even in a limited way, because it oversize the result (reason and / or calculate well or not).

For the measurement of learning, it is essential to take into account what the student must do in a specific area of knowledge, in accordance with the requirements established for their age and school grade. Hence the theoretical and methodological value of the cognitive performance category, which is closely related to the levels of cognitive performance.

In keeping with the exposed line of thought, it was considered useful to re conceptualize levels of cognitive performance of arithmetic problems understanding in primary education. The precision of the corresponding cognitive operations was based on the characterization of the psychic development of primary schoolchildren, the characteristics of the textually of the arithmetic problems and the particularities of the levels of cognitive performance in textual comprehension. Such elements indicate the, growing and dialectical complexity of diverse character.

For the study it was taken as regards the characterization of the ontogenetic evolution of the development of primary school, which considers three moments or essential stages: from 6 to 7 years of age (1st and 2nd grades), 8 to 9 age (3rd and 4th degrees) and 10 to 12 years old (5th and 6th degrees). The foregoing justifies that the proposed cognitive performance levels are conceived as correlations, since - from a systemic perspective - the lower ones are contained in the higher ones; that position explains the recursive nature of the elaboration of meanings during the referred process.

The first level (reproductive) consists of the school's ability to capture local information and some implicit elements (textual sequence, quantitative data, meaning of an expression) in the problem; while the second level (applicative) resides in the possibility of understanding at a more global level the textual content of the problem (inference of meanings by complex relationships, elaboration of critical judgments). For its part, the third level (creative) expresses the highest degree of development reached by a schoolchild, which is characterized by its ability to extrapolate messages to different contexts (experiential situations, mathematical problems and exercises, other texts).

The levels of cognitive performance described have a higher degree of precision and objectivity than those established for textual comprehension, in general (any type of text). This is conditioned by its mobility (relativity), determined by the logical-semantic organization of the textual content of the arithmetic problems. However, they reach a greater degree of uniqueness when considering the psychic development of schoolchildren,

according to educational level; since both factors influence the variability of the degree of complexity of a given cognitive operation.

In line with the above, cognitive operations are specified, belonging to each of the levels of cognitive performance conceived, in correspondence with the moments of ontogenetic development of the primary schoolchild.

First time development: 6 to 7 years (1st and 2nd degrees)

First level (reproductive): recognize places, characters, actions or other information of a local nature; replace a word with a synonym; identify keywords; look up the data of a problem, given explicitly; select the data of a problem without unnecessary data; grasp the given and the requirement (s) in given problems.

Second level (applicative): reformulate expressions; relate a problem to illustrations based on specific elements; paraphrase the situation described in the problem; select the necessary data in simple problems; establish temporary relationships; summarize information by using hyponyms; infer the objective and consequences of the actions; deduce characteristics; assess attitudes.

Third level (creative): generalize the relationships between a problem and its illustration; model graphically given problems; elaborate meanings from part-whole inferences; coordinate equalities to given problems; solve simple problems using graphs or counting; relate problems by the subject that they address and/or practical meanings of the arithmetic calculation operations that are revealed; formulate simple problems from equalities, illustrations, and graphical diagrams.

Second stage of the development: 8 to 10 years (3rd and the 4th grades)

First level (reproductive): order chronologically; reformulate expressions; translate simple expressions from one code to another; identify the arithmetic content that is directly related to the problem and other global information; look up the data of a problem, given implicitly.

Second level (applicative): select necessary data in compound problems; establish complex relationships (partiality, causality, analogy and opposition); develop questions for simple problems; replace information with another logically derived from the text.

Third level (creative): elaborate meanings from chains of simple inferences by relations of partiality, causality and/or analogy; reformulate simple problems; formulate compound problems; solve the problem in different ways; contextualize the meanings of the text to the experiential framework of the scholar.

Third time of the Development: from 11 to 12 years (5th and 6th degrees)

First level (reproductive): paraphrase the problem, omit unnecessary information; translate expressions from one code to another; identify the mathematical and situational context to which the situation described in a problem relates.

Second level (applicative): infer meanings from complex relationships; identify sub problems; represent graphically the situation described; develop graphical diagrams that represent the relationships that occur in the text; assess the attitudes of the characters involved in a described situation.

Third level (creative): elaborate meanings from complex chains of inferences by relations of partiality, causality and/or analogy; formulate compound problems; transform the conditions of the problem to find other ways of solution and/or check the way used; transfer a meaning extracted from the problem to the solution of a new problem, mathematical exercise or task in another subject, to an event in their personal life and/or to some phenomenon in everyday life.

In keeping with the above, for the instrumentation of the categorizing function of cognitive performance levels, it is considered necessary to use percentage analysis. For this, it will be useful to identify the correct answers, by levels of cognitive performance, for the interpretation of the performance of schoolchildren. In addition, the following cut-off points will be taken into account:

- Answer correctly at least 75 % of the questions of the reproductive level.

- Gets the reproductive level and responds at least 60 %, of the questions in the application level.

- Reaches the application level and answers at least 60 %, of the questions of the creative level.

Measuring the cognitive performance of schoolchildren makes it possible to assess, in a more balanced way, the process and the result of understanding arithmetic problems; and identify, with greater accuracy and objectivity, the achievements, the problems and their causes. In the same way, it allows a more integrative evaluation, considering, in a more harmonious way, the unity between the instructive, the educational and the developer.

Consequently, for the diagnosis of understanding arithmetic problems in primary education, indicators and cognitive performance levels proposed in the research were integrated. From its integration, the following ordinal scale was obtained:

Very Low: the student answers correctly and independently less than 75 % of the reproductive questions or requires help to answer correctly, at least that percentage, of questions.

Low: school responds independently at least 75 %, questions of reproductive character.

Medium: the student meets the requirements of the reproductive level and responds with help, at least 60 % of the questions of an applicative nature.

High: The scholar fulfills with the requirements of the reproductive level and answers independently; at least, 60 % of the questions of the applicative character or he can also can answer using at least 60 % of the questions of creative character.

Very High: The scholar meets with the requirements of the applicative level or and responds independently at least 60 %, , questions of creative character.

For the diagnosis of the teaching-learning process of understanding in problem solving, various methods and techniques must be used. The following lines will describe and offer guidance for its instrumentation.

- *Pedagogical Test*

They should be carried out with the objective of measuring the performance of schoolchildren during the understanding of arithmetic problems. For this, instruments must be developed that have at least 10 questions, distributed by levels of cognitive performance as

follows: four from the reproductive level and from the applicative and creative levels, respectively. It should be applied, together with the technique of "thinking aloud" in order to identify the causes of errors and greater precision in the degree of independence of schoolchildren, according to the forms and levels of help required. It is recommended to tabulate the results in tables and / or bar graphs, taking into account the objectives of the study and the sample taken.

- *Observation to classes*

It makes it possible to inquire about the performance of teachers and schoolchildren during the activity under analysis, with emphasis on their limitations and the causes that originate them; as well as in the potentialities that favor the overcoming of limitations. In general, it allows assessing the dynamics of the personal components of the teaching-learning process: teacher, school and group. In line with this, an observation guide should be used, in which the essential elements to be analyzed are taken into account, from the assumption of problem solving as a process of textual understanding.

- *Analysis of the products of the activity*

It is very useful to obtain information on the performance of the teacher and the school. Although it is not limited to this, it is essential that you have as main objects, the lesson plans and the school notebooks. For this, guides must be developed, taking into account the specific objective pursued and the characteristics of the process of understanding arithmetic problems, from the conception assumed in this research.

- *Survey to teachers and principals*

They are useful to obtain and/or contrast information about the preparation of the components of the teaching-learning process. In addition, it allows to identify strengths and limitations, as well as their causes. It must have a consciously elaborated question guide, in correspondence with the objective pursued and the characteristics of the subjects to be surveyed.

- *Interview to school children*

It facilitates obtaining and/or contrasting information on the performance of teachers and schoolchildren in the teaching-learning process, with an emphasis on strengths, limitations and causes. It must be instrumented from a guide of consciously elaborated questions, in correspondence with the objective and the psycho pedagogical particularities of the schoolchildren. The physical conditions of the premises and the dynamics of the activity must guarantee an adequate socio-psychological climate.

DISCUSSION

The main contribution of the research focuses on the recontextualization of dimensions, indicators and levels of cognitive performance of comprehension to the solution of arithmetic problems in primary education. This surpasses the current conceptions, in evaluating not only the product of the understanding process but the process followed by the school, when identifying its achievements, limitations and possible causes; since exercises based on the cognitive operationalization of elaborated and/or selected problems are proposed, taking into account the particularities of their textuality.

The assumption of the solution of arithmetic problems as a process of textual comprehension allows the resignification of the diagnosis of its teaching-learning process. For this, it is necessary to take into account the particularities of its textuality and the

cognitive-affective nature of the textual comprehension process.

Accuracy and cognitive operations of each level of performance of the psychological development of the primary scholar, enables a more accurate and differentiated diagnosis. This gives more objectivity to the information obtained and, therefore, increases the effectiveness of the intervention that is carried out to transform the identified situation. However, it demands for instruments with a higher level of elaboration and processing of more information.

This new perspective also makes feasible the implementation of the postulates about inter subjects relations in the process of teaching and learning, the possible integration of mathematical knowledge and the mother tongue of more internal relations. All this contributes to perfecting the theory and practice of the postulates of developer Didactics.

From the position followed in the investigation, the predominant conception in the teaching of the solution of arithmetic problems of focusing on the understanding of the logical-mathematical relationships that allows satisfying the requirement (s) is overcome. The new proposal allows measuring more harmonious fulfillment of the functions of the problems in the teaching of mathematics, a process in which - influenced by existing contributions -the approach to the education as an aggregate or simply omission predominates.

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