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Systematization of experiences on research in didactics for solving mathematical problems

Sistematización de experiencias sobre la investigación en didáctica de la resolución de problemas matemáticos

Sistematização de experiências sobre a investigação em didática da resolução de problemas matemáticos

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ABSTRACT

Mathematics problem solving is difficult for most students, for this reason the investigators in Didactics of Mathematics constantly works on the construction of theories that help to understand their complex nature, with the purpose of facilitating their teaching learning. Consequently, the objective of this study was to expose the most relevant aspects of research in didactics of solving mathematical problems, obtained in the last twenty years by the Didactic Research Group of Mathematics and Computing, of the University of Oriente, Cuba. The method of systematizing experiences was used, supported by content analysis, interviews and participant observation. The main result was an increase in the knowledge of the theoretical approaches, diagnostic and corroborative means, analytical categories, theoretic - methodological constructs and communication channels, which have shown greater consistency in the investigation of the aforementioned didactics. The above provides research groups with a suitable framework for the study, application and socialization of results, aimed at improving the didactics of solving mathematical problems.

Keywords: mathematics didactic; research groups; problem solving; systematization of experiences.

RESUMEN

La resolución de problemas matemáticos resulta difícil para la mayoría de los estudiantes, por ello los investigadores en Didáctica de la Matemática trabajan constantemente en la construcción de teorías que ayuden a comprender su naturaleza compleja, con el propósito de facilitar su enseñanza y aprendizaje.

Consecuentemente, el objetivo del estudio fue exponer los aspectos más relevantes de la investigación en didáctica de la resolución de problemas matemáticos, obtenidos en los últimos veinte años por el Grupo de Investigación Didáctica de la Matemática y la Computación, de la Universidad de Oriente, Cuba. Se utilizó el método de sistematización de experiencias, apoyado en el análisis de contenidos, entrevistas y la observación participante. El resultado principal fue un incremento en el conocimiento de los enfoques teóricos, medios diagnósticos y corroborativos, categorías analíticas, constructos teórico-metodológicos y canales de comunicación que mayor consistencia han mostrado en la investigación de la referida didáctica. Lo anterior brinda a los grupos de investigación un marco propicio para el estudio, aplicación y socialización de resultados, dirigidos a perfeccionar la didáctica de la resolución de problemas matemáticos.

Palabras clave: didáctica de la matemática; grupos de investigación; resolución de problemas; sistematización de experiencias.

RESUMO

A resolução de problemas matemáticos resulta difícil para a maioria dos estudantes, por isso os investigadores em Didática da Matemática trabalha constantemente na construção de teorias que ajudem a compreender sua natureza complexa, com o propósito de facilitar seu ensino-aprendizagem. Consecuentemente, o objetivo do presente estudo foi expor os aspectos mais relevantes da investigação em didática da resolução de problemas matemáticos, obtidos nos últimos vinte anos pelo Grupo de Investigação Didática da Matemática e a Computação, da Universidade do Oriente, Cuba. Utilizou-se o método de sistematização de experiências, apoiado na análise de

conteúdo, entrevistas e a observação participante. O resultado principal foi um incremento no conhecimento dos enfoques teóricos, meios diagnósticos e corroborativos, categorias analíticas, constructos teórico-metodológicos e canais de comunicação, que maior consistência mostrou na investigação da referida didáctica. O anterior brinda aos grupos de investigação um marco propício para o estudo, aplicação e socialização de resultados, dirigidos a aperfeiçoar a didáctica da resolução de problemas matemáticos.

Palavras chaves: didáctica da matemática; grupos de investigação; resolução de problemas; sistematização de experiências.

INTRODUCTION

The study of Mathematics always has been a stressful process for the students, but learning should not be a sacrifice; Therefore, professors and researchers from around the world seek, incessantly, ways to transform in satisfaction the anxiety that their study generates, which does not mean the disappearance of effort and perseverance; but, by the contrary, the abundance of effective reasoning and accepted in a consciously way (Alonso, Gorina, Churches & Alvarez, 2018).

And that this that transformer endeavor becomes possible because mathematics has the potential to transcend the limits of the teaching content, and influence the development of logical thinking, so it is necessary to develop those cognitive skills, essential to learn and apply efficiently mathematical knowledge to problem solving. So while

students solve problems, they perceive the usefulness of this science to the world that surrounds them, which will allow them to develop motivations, attitudes and thoughts that Fortress learning (Alonso & Martinez, 2003).

In this direction, it should be noted that the didactic approach to teaching Mathematics through problem solving has more and more followers. This is because the advantageous for learning of the activities to be developed by students when they have to solve a problem, because it must mobilized mental capacity to recover the knowledge they have, work with objects present, to develop analogies with problems already solved, to exercise creativity, to reflection about its thought process to improve it, to transfer activities to other aspects of mental work, to acquire confidence and security in himself, enjoy himself with his own mental activity and prepare himself to face new problems (Alonso, Gorina, Iglesias & Álvarez, 2018).

A deepening of the literature on the subject allows us to verify that there are numerous works that have been oriented to the improvement of the didactics of mathematical problem solving, providing valuable theoretical and methodological results that have helped to have a more complete vision of the teaching of the referred process (Shoenfeld, 1992; Alonso, 2001; Polya, 2004; Álvarez *et al.*, 2012; Alonso, Gorina, Iglesias & Álvarez, 2018; Álvarez *et al.*, 2018; Iglesias *et al.*, 2019).

However, despite the existing interest in the study of this subject and of the valuable results that have been obtained, it is still observed several difficulties in the learning of mathematical problem solving by students,

manifested mainly in their limited success in implementing general, heuristics and metacognitive strategies during the resolute process (Alonso, Gorina, Churches & Álvarez, 2018).

In the case of Cuba, several universities have taken sides in this laudable effort to conduct research in Mathematics Education, being the most representative events in study centers, lines and projects research as the main organizational forms of scientific work. There have also been isolated researchers who have made important contributions to solving mathematical problems. However, these universities do not have promoted sufficiently creation of research groups that drive the scientific collaborative work on this important subject.

It should be noted that research groups have a complex degree of interaction, characterized by the performance of collective tasks that require collaboration, coordination and communication, encouraging their members to achieve continuous development. They, moreover, facilitate that new generations of scientists incorporate and internalize the ways to investigate, values and regularities of each scientific discipline, as well as contribute to their development in a stable way.

The need for searching effective ways to teach math problems from a collaborative perspective, led to several professors d Research Group Teaching Mathematics and Computer (GIDMAC), University of Oriente (UO), Cuba, to develop research on this subject, product of which various theoretical and methodological constructs have been developed and applied that have made it possible to explain and intervene in the didactics of mathematical problem solving during the last twenty years, in

various university careers belonging to the Sciences Exact, Natural and Technical.

However, it was decided to make a pause in the investigative path and critically question the experience the GIDMAC to reach conclusions that allowed building new proposals from that experience. Indeed, the aim of the present study was to present the most relevant aspects of research in teaching of mathematical problem solving, obtained in the last twenty years by the aforementioned group of University of Oriente .

The importance of managed this goal is that, from the systematization of the experience developed, IT could get learning about what has been done, to produce new knowledge and socializes the main results , as a basis for encouraging the creation and development of new research groups that have among their priorities the research in the teaching of mathematical problem solving . It could ultimately improve the learning of Mathematics and its application for the benefit of the sustainable development of society.

MATERIALS AND METHODS

To perform the present study, the method of systematization of experiences was used (Expósito & Gonzalez, 2017; Ramos, Rhea, Pla & Abreu, 2018), which is based on the critical paradigm, hence enabling use critical interpretation as a way to build knowledge.

The systematization team was made up of six GIDMAC doctors and a visiting doctor (specialist in teaching

mathematical problem solving). The first task was to study in detail the method of Systematization of Experiences. Once discussed and understood every aspect of it, it went on to implement the methodology of systematization and collect all publications, papers, theses and other documents relating to the GIDMAC related to the subject, belonging to the predefined stage.

The methodology of the systematization of experiences method was followed , defining as an object: the didactics of solving mathematical problems; in which it has re dropped the investigative action developed on the subject by members of the GIDMAC; it was reflected on the approaches employed, theoretical and methodological constructs applied for the transformation, the main analytical categories used, the means of diagnostics and for corroboration of results used, the channels for communication of research results, as well as strengths and weaknesses evidenced.

The experience to be systematized was delimited as: investigation of didactics of mathematical problem solving carried out by members of GIDMAC during the last twenty years. Here it should be noted that the GIDMAC has 17 years of existence, but the first doctoral thesis used in this systematization was developed by the founder of it five years earlier.

The objective of the project was to learn, produce knowledge, build proposals and socialize the results of the systematization. Based on this objective, five systematization axes, which allowed problematizing the experience analyzed for critical interpretation, were required. They were:

- Main focus used in the investigation of the teaching of mathematical problem solving.
 - Types of theoretical and methodological constructs provided to improve the didactics of mathematical problem solving.
 - Main analytical categories used to support the theoretical and methodological contributions made.
 - Diagnostics means and corroborative used for exploration and evaluation of the results of the research.
 - Channels used to generalize, communicate and transfer the main scientific-investigative results.
- Develop a documentary review of the research projects carried out in the pre-established period, the doctoral and master's theses tutored and defended by GIDMAC members, the published scientific articles, the papers presented at scientific events, the participation in courts of teaching categories of Assistant professors and Associate professors, the activities of internationalization, work in multidisciplinary teams, the teaching of postgraduate education (postgraduate courses, trainings, workshops, seminars scientists, consultants, etc), the teaching of undergraduate teaching (address diploma works) and the awards obtained for the research work carried out.

Subsequently, the following problematic questions were formulated, to critically question the experience under analysis: what can you conclude about the suitability of theories that, from the epistemological, psychological and educational approaches, have sustained the investigation of the teaching-learning process of solving mathematical problems, in the experience under study? What kind of theoretical and methodological constructs have been used to address the process and how they are articulated in each investigation? What are the main categories used to substantiate the theoretical and methodological contributions made? Have it been efficient the means that have been used to carry out diagnostics and research results? Have it been relevant the channels used to generalize, communicate and transfer scientific research results?

Finally, to prepare for the orderly reconstruction of the experience, it was necessary to develop the work plan that was projected, which included the following activities:

- To develop a workshop with members of the team and specialist Just invited, to discuss the orderly reconstruction of the experience.
- To redact the preliminary report on the orderly reconstruction of experience.
- To discuss the preliminary report with members of the GIDMAC to improve and enrich it with aspects that might have be overlooked.
- To elaborate the final report of the orderly reconstruction of experience.

RESULTS

Orderly reconstruction of experience

The beginning of the experience was fixed in the preparation of the doctoral thesis entitled "The mathematical problem solving. An educational alternative focused on the representation" (Alonso, 2001),

which delved into the representation that of the mathematical problem and the process of resolution is based on students of the Bachelor in Mathematics, providing an operational structure for skill *represent mathematical problems* and base a classification of the representations of mathematical problems based on the different objects of the representative world.

The aforementioned doctoral thesis obtained a novel conception of the resolution process, considering it as an evolution of the representations of the problem, with two fundamental moments: the conception of an initial representation of the problem and the progressive generation of a series of representations that lead to the solution.

These theoretical results were implemented in a model of the process of solving mathematical problems and were the basis for the development of a teaching strategy that facilitated the conduct of the same. The application of the didactic strategy was carried out for more than fifteen courses, in the first year of the Bachelor's degree in Mathematics at the UO, for which it was used as a textbook *How to be successful when solving mathematical problems?*, by the author of the thesis and Dr. C. Henry González, published by Editorial Visión Creativa (Alonso & González, 2003). The results of the thesis generated numerous publications, presentations at scientific events and the development of postgraduate courses.

Later, this doctoral thesis was followed by other investigations that generated five master's thesis and three doctoral theses on the subject, which shared approaches, categories, types of theoretical and methodological constructs and diagnostic means:

- *The PLEC ++: An interactive software proposal for solving optimization problems.* It studied the limited functionality of existing optimization computer programs for teaching and learning subjects in the discipline of Optimization in the Bachelor of Mathematics career and provided one with enough interactivity to activate the teaching-learning process of the aforementioned discipline, which also integrates the contents of the subjects Mathematical Optimization I and Mathematical Optimization II and that serves as a didactic means for the development such process. This *software* has been introduced in the Bachelor's degree in Mathematics at the UO and in several postgraduate courses in Cuba, Bolivia, Mexico and Venezuela (Gorina *et al.*, 2017).
- *Valorative systematization of solving mathematical problems. An educational proposal.* Shortcomings in the system of beliefs about mathematical problem solving in students of grade 12 in two senior high schools in the city of Santiago de Cuba: "Rafael María de Mendive" and "Cuqui Bosch" were diagnosed. It was provided a strategy to guide the dynamics of the evaluative process systematization of mathematical problem solving. The results of this research were published and presented at scientific events (Gorina *et al.*, 2017).
- *Dynamics of inductive reasoning in solving mathematical problems. A didactic proposal.* To deepen in the inadequacies in the reasoning of the mathematical problems of students in the first year of Bachelor in Mathematics. A strategy was

provided to drive the dynamics of the inductive reasoning process in the problem solving process. The results of the thesis generated numerous publications and presentations at scientific events (Álvarez, Alonso & Gorina, 2012).

- *Educational strategy for the formation of the value of perseverance in solving mathematical problems. It was analyzed, in a sample of students who have overcome grade 12 and preparing to attend college, how they solved the mathematic problems, the interest shown in developing this work and effort he puts into it. An educational strategy was provided to guide the development of the dynamics of the process of formation of the value of perseverance in solving mathematical problems. The results of the thesis were introduced in undergraduate and graduate courses, publications, and presentations at scientific events (Alonso, Gorina & Santies teban, 2018).*
- *Logical-algorithmic dynamics of the process of solving computer programming problems. It diagnosed an insufficient interpretation of problem situations, which limited the efficiency in solving computational problems. This research is considered a generalization of the previous ones , because it managed to establish the double modeling, mathematical and computational, which must experience a problem situation on the algorithmic path that leads to its solution. He contributed a model of the logical-algorithmic dynamics of the process of solving computer programming problems, which contains as first dimensions the mathematical representation and algorithmization of the problem. It also provided a system of didactic procedures to conduct the teaching of said algorithmization. Their results were introduced in two computer science majors at the UO and served as content for various postgraduate courses, publications, and presentations at scientific events (Salgado, Alonso & Gorina, 2019).*
- *Basic system of skills for computational algorithmization. He gave continuity to the doctoral thesis "Logical-algorithmic dynamics of the process of solving computer programming problems". It provided a system of skills that guides teachers for their training and favors the improvement of the teaching-learning process of algorithmization (mathematical and computational), to be developed by programming subjects. The research results were disclosed through postgraduate courses, publications, and presentations at scientific events (Alonso, Salgado & Blanco, 2020).*
- *Interdisciplinary dynamics of Differential and Integral Calculus focused on the projective-structural aspect of Civil Engineering. He diagnosed an insufficient appropriation of mathematical content, in relation to its application to solving problems that are based on certain mathematical methods. He provided a model and a didactic strategy to conduct such interdisciplinary dynamics, as well as a methodology for the elaboration of a matrix of articulation of nodal content. The*

results obtained in this doctoral thesis were applied in the aforementioned career, of the UO and disseminated through publications and presentations at scientific events (Iglesias, Alonso & Gorina, 2019).

- *Dynamics of inductive-deductive reasoning for the resolution of mathematical demonstration problems in mathematical profile careers.* He gave continuity to the master's thesis "Dynamics of inductive reasoning in solving mathematical problems. A didactic proposal". He provided a model of the dynamics of the teaching-learning process of inductive-deductive reasoning in the resolution of mathematical demonstration problems in mathematical profile careers, which became a method and a system of didactic procedures to guide teachers and students about how to put into practice the teaching-learning of said reasoning in these careers. Their contributions were introduced in the Bachelor in Mathematics from the UO and will I spread Eron through articles and presentations at scientific events (Alvarez, Alonso & Gorina, 2018).

Regardless of the arguments presented above, the GIDMAC has always had a line of research associated with the teaching of solving statistical problems in pre - university education and the university (undergraduate and graduate). Since this have derived several courses and publications, issues on a statistical competition in the training process research scientist doctors in Pedagogical Sciences, the data processing logic and elementary statistics in pre - university education and resolution problems to promote the formation

of statistical thinking, reasoning and culture at the university level (Gorina & Alonso, 2014).

Analysis and critical interpretation of the experience

From each of the five axes of systematization defined to critically reflect on the experience, and based on the discussion of each question problematical, is LLEV or out a deepening in the aspects Theophilus rich, conceptual, that specific methodologies cal, technological and ethical, linked to the diversity and complexity of the research contents to associates to each axis.

The analysis of the *first axis of systematization* made it possible to specify that the research carried out was based on two epistemological approaches that constitute systemic variants. The first was on the Theory Holistic - configurational (Fuente s & Alvarez, 2017), which she allowed or recognize the teaching of problem solving mathematical, as a system of conscious processes, holistic and dialectical nature, facilitates NDO its modeling didactics Through the Holistic-Dialectical method, which inherits the presuppositions and operates with the analytical categories of the aforementioned theory (configurations, dimensions, links, systems of relationships and regularities).

The second approach used was the Theory of Systems and, so special, its method Systemic Structure Functional, which facilitated the foundation and structure of teaching tools created to intervene in the process of teaching and learning problem solving mathematical to starting from analytical categories that distinguish systems such as recursion,

synergy, entropy, homeostasis, self-development, among others. In the structure of these instruments, the objective, the actions to be carried out by the teachers and the students, the evaluation criteria and the achievement patterns were also defined, maintaining a close relationship with the categories of the modeling carried out.

Also from the psychological predominant or use the cognitive approach to processing information from Cognitive Psychology Contemporary. It connoted, in addition, the concept of Learning Meaningful (Ausubel, 1983), framed in the constructivism and the Focus Historical Cultural, supported by Vygotsky (1995). It took advantage or postulated about the need to help develop skills to learn to think of efficiently, regardless of the instructional context; and its proposal consider to the student as an active processor of information and teaching as a stakeholder guide on teaching, effectively, knowledge, cognitive, metacognitive and author regulations, always based on prior knowledge of the student and experiences and schemes. The postulate about the formation of an internal organization of information in schemes and rules was also considered, which is re-elaborated based on exchanges with the outside to interpret, re-signify and dynamically configure reality.

From the Cultural Historical Approach, Vygotsky (1995) to ok the thesis on the origin of higher psychic functions and the role of mediation in learning, which ponders the interactive nature of the teaching-learning process developer, through his law of the double formation of the higher psychic processes. This postulate is consistent with the foundation that is made of the resolution of mathematical

problems, which is a perfect model of higher psychological function or complex mental process, since in it there is an intertwining with other psychic functions: language, abstract thinking or reasoning. (Induction, deduction), etc.

As approaches Didactic I prevailed the subgested by Polya (2004), mainly general method of solving mathematical problems, which has been considered starting paradigm for research in the field of teaching and learning process of resolution of math problems. This method has been materialized in the provided scientific results by the GIDMAC, to the created theoretical and practical constructs to influence the above teaching-learning from particular edges of the same, as represents the problem mathematical and process resolution, inductive-deductive reasoning, algorithmization, interdisciplinary work, among others.

Schoenfeld's work has also been repeatedly used, especially his proposal for the necessary resources for solving mathematical problems (Schoenfeld, 1992). This result allowed deepen the belief system of students on solving mathematical problems and the formation of values, mainly in perseverance on the resolver i to mathematical problems.

As a conclusion of this first axis of systematization, it is considered that, although other approaches, related to the subject investigated, could be used, those considered as main have been effective in supporting the contributions made.

For the second axis systematization, based on constructs theory and methodology employed and their articulation, was observed in the case of theoretical definitions, have prevailed teaching models and regularities. Skill systems,

guidelines, theoretical-methodological bases and a content articulation matrix have also been provided. Inside methodological have been more frequent systems teaching methods, teaching strategies, educational strategies, methods, methodologies and textbooks.

The permit analysis of this axis to noted that various constructs used were theoretical and methodological; highlighting the direct correspondence remained between theoretical contribution and practical (methodological) the concrete. In most cases, the theoretical contribution was made under the Holistic- Configurationally approach and the practical one was supported by the Functional Structural Systemic, taking advantage of the dimensions defined in the first to structure the phases, stages or procedures of the second. This allowed to give coherence to both contributions and to facilitate the practical materialization of the relationships revealed from the theoretical point of view.

Another interesting aspect was the fact that teaching and educational models and theoretical and methodological bring us basis to us, were not limited to the representation of what already exists and what is being done in the teaching - learning under study but that, in each new elements are introduced which addresses run the necessary transformation of the process, from the various fields of action were addressed. Hence the practical constructs plasm as in the objectives, actions and the methods to be used by teachers and students to achieve these transformations.

These practical constructs also contain evaluative criteria and achievement patterns to measure the results that teachers and students obtain throughout the training

process. In all cases, they have been applied and specialists and experts have been consulted about their relevance and feasibility of application, obtaining important evaluations that have allowed their improvement.

To conclude this second axis of systematization is considered that the theoretical and methodological constructs provided distinguished for being articulated to each other, within the framework of each investigation, being relevant and applicable to the teaching-learning for those who they were designed and admitting generalizations in most cases.

The third axis of systematization, concerning the analytical categories used to substantiate the contributions made, is the richest. His analysis allowed us to observe that there are several nuclei of common categories. The first or of all or s is whose center is the definition of *mathematical problem* provided by Alonso (2001), which has been used in other investigations. This is because it facilitates the investigation of the didactics of mathematical problem solving, by making explicit the type of information provided by the problem and its structure, which allows orienting the student in the analysis and resolution of these problems.

- It has been complemented with a classification of mathematical problems, among which two investigations relating to *mathematical demonstration problems stand out* (Álvarez, Alonso & Gorina, 2012; Álvarez, Alonso & Gorina, 2018). In some cases, this definition has been expanded (Salgado, Alonso & Gorina, 2019; Alonso, Salgado & Blanco, 2020), starting from considering a problem *situation* in terms of real objects and relationships,

which are converted into a mathematical problem. In the same way, it has started from a projective-structural problem, associated with the way of acting of the civil engineer and the definition of a mathematical problem has been generalized (Iglesias, Alonso & Gorina, 2019).

Another category very close to that of a mathematical problem, which has been systematically used in the research under analysis, is that of representation. The internal and external representation of the mathematical problem and its resolution process have been considered. As in the previous case, representations of a problem situation have been considered (in terms of real objects and relationships), which has been led to mathematical representations and, later, to computational representations, the latter being considered as a pseudo-coded generalization of a mathematical representation. Syntactic and semantic evaluations of a representation have also been introduced.

A second group of categories is the one that is related to the necessary resources to approach the process of solving mathematical problems. In this case we have worked with specific knowledge, math abilities, strategies (heuristics and metacognitive), the algorithmization (first and then computational mathematics), problematization, contextualization and didactic transposition of mathematical content. In addition, interdisciplinary work has been introduced, considering nodes and matrices of articulation of mathematical content with other professional content, revealing the functionality of the former.

In the particular case of the mathematical demonstration problem, in addition to the above categories, inductive and deductive reasoning, mathematical conjecture, inductive validation of mathematical conjectures and deductive demonstration of mathematical conjectures were used.

In addition, they have been considered within the categories employed, those that allow the formation of dynamic competitions and value is. Such is the case of cognitive and affective stimulation, systematization and mathematical-solver socialization and the value of solver persistence.

We conclude about the third axis of systematization, that the categories presented have been of inestimable value to develop these investigations, some of which have been redefined by researchers from the GIDMAC own.

This and deepens the fourth systematization axis, comprising diagnostic means that allow exploring the processes and phenomena of interest; as well as, determine its insufficiencies and tendencies. This determination is made based on data and facts systematically collected and sorted, allowing better judge what is aconteciece NDO. It is also used to corroborate research results.

The aforementioned deepening allowed to identify as a general pattern of the analyzed investigations, that they are carried out in three moments. One or at the beginning of the process, with the aim of basing the objectivity of the research problem, OTR or when you have already defined the scope to characterize the current state of this and finally, when practical tools designed apply. In the latter case, the temporal space of analysis has changed and

the pedagogical quasi-experiment has been used fundamentally.

The diagnostic process has been carried out using as main media: document analysis (programs, curricula, student records, textbooks, results evaluative, etc), surveys (teachers, students, managers, employers, etc), written tests with selected problems, observation of student performance during problem solving, observation of teaching activities (classes, exams, etc). All have been preceded by the definition of indicators to be used, the measurement scales, the design of the silver way to be seated information and pilot studies, among other things necessary to ensure the reliability and validity of the information.

After collecting the data, they have been processed, using quantitative and qualitative methods, generally articulated, to strengthen the conclusions drawn. The triangulation of the results obtained with various diagnostic means has been another element that has contributed to enhancing the quality of these conclusions.

The corroborations of the contributions have been made using the opinion of specialists (in socialization workshops) and experts (in consultations), who have valued its relevance and feasibility of application, then the didactic instrument provided has been exemplified by means of a selected problem and it has passed to its practical application in all cases (through its introduction in the teaching- learning process of some subjects). All this has been complemented with the observation of the performance of the students and surveys to them on the quality of the classes and the satisfaction with what they have learned; as well as with interviews with teachers, and the development of some pedagogical experiments. In all cases, the results

obtained have served to improve the contributions.

It is concluded for the fourth axis of systematization that the means used to diagnose and corroborate the results have been numerous and varied, which has allowed guaranteeing the adequate level of quality of the investigations carried out by the GIDMAC. An interesting practice is that of the double diagnosis carried out, since it is considered that they are two different moments of the investigative process: at the beginning, when it is necessary to establish the objectivity of the research problem and, later, when the design of the investigation has been completed. Referred research and works on the foundation of its object and its field of action. However, the most remarkable thing about this axis is the use of qualitative and quantitative methods, in an articulated way, to process the information extracted from the diagnosis, since the GIDMAC has specialized in this type of methodological articulation through the application of mixed methods.

Finally, in the fifth axis systematization relevance of channels used to generalize analyzed, communicate and transfer scientific research results and summarized the generalization was carried out by: to own sequence temporary, in the execution of research projects, achieving a theoretical-methodological reconstruction by carrying out new research at GIDMAC; in the generalized introduction of results in undergraduate and postgraduate teaching, through computational tools, teaching materials and didactic instruments and the adaptation and updating of curricula; and the work of diplomas, to the real problems been systematic updating of members of the tribunal categories of teachers and professors who presented it.

In turn, it has developed communication through the publication of books and scientific articles, the development of guidelines to guide the methodological work, teaching materials, and has involved do events scientists, committees or scientific magazine publishers, academic and social networks, informative activities, etc.

For its part, the transfer has been carried out through methodological work, the delivery of postgraduate teaching (courses, trainings, specialized conferences and academic training), internationalization work and collaborative work and joint research, international projects, national and international collaboration stays, among others.

A road that has enabled corroborate the relevance and impact achieved in generalization, communicates and transference of research scientific results generated by the GIDMAC, from research in the teaching of mathematical problem solving, has been obtaining premiums: the Pablo Miquel and Merino (2013 and 2019), awarded s by the Cuban Society of Mathematics and Computer Science (SCMC) in the category "Teaching Mathematics", two provincial awards granted by the Ministry of Science, Technology and Environment Cuba environment (CITMA) and numerous awards research granted by the University of Oriente, Cuba.

In conclusion of this fifth axis of systematization, highlights the diversity of channels used to generalize, communicate and transfer new contributions theoretical methodological years from research on teaching of mathematical problem solving, which have been Enter two to perfect undergraduate and graduate teaching, and have been disseminated through publications and scientific events, as well as through

collaborative work and internationalization. However, the most outstanding feature in this area has been the development of the digital bibliographic identity of the GIDMAC members, which has favored the achievement of greater visibility of their scientific results and the obtaining of research awards.

DISCUSSION

They prevail in the world three basic approaches to the didactics of solving mathematical problems:

- The resolution of a problem to justify the need to study a certain content.
- Solving problems that illustrate the application of content that has been explained s abstract.
- Problem solving across all topics and classes, so that students have the opportunity to appropriate resolution patterns, construct conjectures, and expose and defend their ideas, among other activities.

Precisely, the latter approach is the one that has Asum gone by GIDMAC to carry out teaching through problem solving, which allows achieve an active and meaningful learning because it puts the emphasis on the thought processes and learning, taking mathematical content as a field of systematization, to build effective ways of thinking.

In the case of Cuba, there are results of research is independent who have made valuable contributions to the didactic of solving mathematical problems, proposing constructs theoretical and methodological directed to perfect this process and contribute to the development of the capacity to formulate and solve problems. However, there are

limitations regarding collective work on this issue.

It should be noted that in the national context there is little information on recognized research groups, whose object is the Didactics of Mathematics. In this direction, one important place it occupied the BETA group, which led Dr. Herminia Hernandez Fernandez, the which had among its main lines investigation, the foundation of a system mathematical abilities and the mathematical oakmas (Alonso, 2001).

From the exchange with researchers in scientific events and by communicating digitally and direct, it could be known that besides the GIDMAC currently exist in the country's least three research groups in teaching Mathematic, the running stability, but in an isolated way, without achieving the necessary investigative exchange between them, which would allow them to advance more quickly and raise the quality of their results.

One of these groups is called Educational Mathematics, which is based at the University of Camagüey and is directed by Dr. Olga Lidia Pérez González. This advances several lines within the Didactics of mathematics related skills training and math skills, development of mathematical thinking from an engineering perspective and mathematical didactic improvement of teachers, among others.

The second group of Educational Mathematics carries out its investigations from the Technological University of Havana (CUJAE), directed by Dr. Raúl Delgado Rubí; and this case, with lines attached to the teaching of Mathematic with the use of technology, improvement of teaching Linear Algebra in engineering careers and training of research skills from mathematics.

The third group has been called Teaching of Mathematics and belongs to the University of Informatics Sciences. It is coordinated by the MSc. Alexander Rodriguez Rabelo and its line of research improving the teaching and learning of Mathematic careers in computer engineering profile.

As can be seen, none of these three groups has considered the resolution of mathematical problems as one of their research lines. Hence, the creation of new groups is considered necessary, which form important functional units and promote collaborative scientific work in all its lines and, especially, in the Didactics of solving mathematical problems. In addition, it is valued that existing groups should continue to perfect the strategies to develop their digital bibliographic identity, in order to increase their national and international visibility, in order to achieve a higher level of collaboration with other groups and researchers.

It is recognized that the systematization of experience made in this study could be biased selection regarding the information to be present or because it is not easy in a few pages translate the results most relevant research on teaching of mathematical problem solving, that over 20 years have been obtained within the framework of GIDMAC.

It had been guiding, being possible to most level of detail on the results presented. However, interested readers can consult the literature cited as an alternative to deepen these specificities.

It should be noted that the dynamics of work usually experienced in higher education does not favor that make one pause in the investigative path to critically question the experience. However, thanks to this

pause, the members of the aforementioned group have reached valuable conclusions that permit them to reorient their group goals and make a new strategic programming, in order to achieve new goals demanded by today's society and the own professional development.

Regardless of the possible biases that have been generated when presenting the results of the systematization of experience, it is considered that the objective of the article has been achieved, as is exposed the most important aspects of research in didactics solving mathematical problems, obtained in the last twenty years by GIDMAC. Which can be summarized below:

- The research ones made were based on a system perspective provided by the theory Holístico- configurational and the Theory Systems. From the psychological, she predominated the use of the Theory of Meaningful Learning (Ausubel, 1983) and the Focus Historical Cultural (Vygotsky, 1995). Meanwhile, the didactic approaches that were most used were those provided by the general method of solving mathematical problems, provided by (Polya, 2004) and the necessary resources for such resolution, proposed by Schoenfeld (1992).
- The theoretical and methodological constructs contributed to the didactics of mathematical problem solving have been distinguished by being articulated with each other, within the framework of each investigation carried out, being pertinent and applicable to the teaching-learning processes for which they were designed and admitting generalizations, in most cases, to various contexts.
- All analytical categories used in research have been of inestimable value for their proper development, some have been used systematically and the researchers themselves have proposed others, in order to achieve a better didactic foundation for solving mathematical problems.
- The means used for the diagnosis of problem situations and the corroboration of the results of the investigations carried out have been varied, which has allowed guaranteeing high levels of validity and reliability with respect to the information managed, being key to this the articulated use of mixed methods research.
- The channels used to generalize, communicate and transfer the research results have been diverse and highly functional, and have helped improve teaching undergraduate and graduate students as well as to disseminate the results through publications, events and scientific work collaborative academic network, which has increased its visibility and impacts, favoring obtaining important awards.

In short, through the systematization of the experience presented, she was able to reveal a subject knowledge to be used by other researchers in the establishment or development of research groups in Mathematics Education, which have among their priorities didactics resolution math problems. It is considered that such an initiative could ultimately improve the learning of Mathematics and its application in favor of the sustainable development of society.

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