



Original article

The teaching of logarithmic equations. Preparation for college entrance examinations

Procedimiento para la enseñanza de las ecuaciones logarítmicas en la preparación de los exámenes de ingreso a la universidad

O ensino das equações logarítmicas: Preparação para os exames de admissão à universidade

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ABSTRACT

In the process of teaching and learning Mathematics at the pre-university level, difficulties related to logarithmic equations

have been identified among students. Addressing this issue is crucial to ensure a solid foundation. The objective of this article is to present a specific procedure to promote effective learning of logarithmic equations in pre-university education. The study was conducted between 2020 and 2024 at the "Federico Engels" Pre-University Institute in Pinar del Río. The research employed theoretical methods such as historical-logical analysis, along with empirical methods including document analysis, interviews, and classroom observations. The main outcome was a methodological procedure specifically designed to tackle logarithmic equations in the pre-university context, emphasizing the use of different content variants and individualized attention to students. The procedure was based on prior diagnosis and a preventive approach. The conclusions highlight common challenges faced by students when solving logarithmic equations, emphasizing the importance of consistent practice and application of learned properties.

Keywords: Teaching; logarithmic equations; exam; prevention.

RESUMEN

En el proceso de enseñanza-aprendizaje de la Matemática en el nivel preuniversitario, se han identificado dificultades en el aprendizaje de las estudiantes relacionadas con las ecuaciones logarítmicas. Abordar esta problemática es crucial para garantizar una sólida preparación. El objetivo de este artículo es presentar un procedimiento específico para fomentar el aprendizaje efectivo de las ecuaciones logarítmicas en la enseñanza preuniversitaria. El estudio se llevó a cabo entre los años 2020 y 2024 en el IPU "Federico Engels" de Pinar del Río. Se utilizaron métodos teóricos como el histórico-lógico, de conjunto con el análisis de documentos, entrevistas y observación, como empíricos, aplicados. El principal resultado estuvo en un procedimiento metodológico diseñado específicamente para abordar las ecuaciones logarítmicas en el contexto preuniversitario que pone

ênfasis en la utilización de variantes del contenido y en la atención a las diferencias individuales de los estudiantes. El procedimiento se basó en un diagnóstico previo y en un enfoque preventivo. Las conclusiones estuvieron en la reflexión sobre los desafíos comunes que enfrentan los estudiantes al resolver ecuaciones logarítmicas, la importancia de la práctica constante y la aplicación de las propiedades aprendidas.

Palabras clave: Enseñanza; ecuaciones logarítmicas; examen; prevención.

RESUMO

No processo de ensino-aprendizagem da Matemática no nível pré-universitário, foram identificadas dificuldades relacionadas às equações logarítmicas entre os estudantes. Abordar essa questão é crucial para garantir uma base sólida. O objetivo deste artigo é apresentar um procedimento específico para promover a aprendizagem eficaz de equações logarítmicas no ensino pré-universitário. O estudo foi realizado entre 2020 e 2024 no Instituto Pré-Universitário "Federico Engels" em Pinar del Río. A pesquisa utilizou métodos teóricos, como análise histórico-lógica, juntamente com métodos empíricos, incluindo análise de documentos, entrevistas e observações em sala de aula. O principal resultado foi um procedimento metodológico projetado especificamente para abordar equações logarítmicas no contexto pré-universitário, enfatizando o uso de diferentes variantes de conteúdo e atenção individualizada aos estudantes. O procedimento baseou-se em diagnóstica prévio e em uma abordagem preventiva. As conclusões destacam os desafios comuns enfrentados pelos estudantes ao resolver equações logarítmicas, enfatizando a importância da prática consistente e aplicação das propriedades aprendidas.

Palavras-chave: Ensino; equações logarítmicas; exame; prevenção.

INTRODUCTION

Pre-university education has the mission of achieving a comprehensive general culture where the teaching-learning process is its main way, so preparing students from the contents of the subjects, specifically those of Mathematics, allows them better preparation in their studies. of the higher level for all the logical operations of thought that he develops and as a result future success in his profession. A correct, widely generalized criterion is that they promote logical thinking, problem solving and informed decision making, essential skills for daily life and professional success (Moreno et al., 2023).

The essential objective of teaching Mathematics is the students' understanding of its contents, which will allow them to develop skills and obtain new knowledge based on previous knowledge from previous grades.

For Díaz et al. (2014):

Learning difficulties in Mathematics are those that affect the development of skills related to mathematics. To be diagnosed as such, they must affect school performance, as well as generate problems in terms of social, personal, school, family, work or community adaptation. (p.2).

Teachers who teach the subject at this level of teaching are required to be able to direct the teaching-learning process creatively and facilitate interaction since students learn more effectively when they do so cooperatively (Tigse , 2019). This enables the graduate to obtain the necessary knowledge and develop the skills they will need to successfully face mathematical content at other educational levels, as well as for its application throughout life.

For González (2022), the teaching of Mathematics, for the Cuban nation, rests on solid scientific bases, with a marked orientation towards the development of the personality of the students and with the aspiration of achieving developmental teaching, however the author, states that "there is a certain consensus in the idea that the failure or success of students in the Higher Education Entrance Tests depends fundamentally on the quality of the teachers' performance" (p.2)

Learning necessarily involves cognitive ruptures, accommodations, changes in implicit models (conceptions), languages, and cognitive systems. However, some ideas resist and persist, say Caronía et al. (2014). Seen from this perspective, the analysis of achievement or failure in mathematics demonstrates that this is a multifactorial phenomenon, which often has relationships of interaction and mutual interdependence in which some variables can be determined that can be significantly related to learning or can predict failure or difficulties in school mathematical learning, among which the relationship between those of a cognitive-affective nature and the level of achievement achieved by students in the teaching-learning process of mathematical content stands out (Gamal Cerda et al. ., 2017).

On the other hand, Aliaga et al. (2015) consider that it is necessary for the teachers in charge of this preparation to have mastery, first of all, of the diagnosis of each of their students, in addition to providing the corresponding authorities with the attention and preparation of the teachers in charge of developing this program in each of them. the centers. It is therefore important to place emphasis on the various ways in which the content is presented, as well as the level of development achieved by each of the students, which constitutes an element of motivation for studying the subject and achieving higher levels. Learning. The preparation of the tasks in each of the classes must be directed to this end, that is, to the establishment of the different ways in which the content can be presented and the

levels of development reached by each of the students.

Attention to individual differences, taking preventive work as the center and the variety of ways in which logarithmic equations can be presented in Mathematics classes, plays a decisive role in achieving the objectives proposed in pre-university, given its possibilities and potentialities.

However, the teaching-learning process of pre-university Mathematics shows difficulties in the content related to logarithmic equations in which disinterest in learning and lack of motivation for learning are manifested, generating low academic performance, confirmed in the results of the exams in pre-university education, in the preparation exams for the entrance test that are carried out in this education and the results of the entrance exams in the elements related to logarithmic equations.

The article is aimed at presenting a procedure for promoting the learning of logarithmic equations in pre-university education developed between 2020 and 2024, at the IPU "Federico Engels" in Pinar del Río.

MATERIALS AND METHODS

In the context of the study carried out at the IPU "Federico Engels" in Pinar del Río, an educational diagnosis was applied to 80 11th grade students at the beginning of the 2022-2023 school year and during the course of the didactic unit on LOGARITHMIC AND EXPONENTIAL FUNCTIONS . The main objective was to verify the level of development achieved by the students in these specific contents.

As a fundamental theoretical method, the historical-logical method allowed the analysis of the main references about the process of solving logarithmic equations, with emphasis on the difficulties that students have regarding the preceding content and the impact on the solution process of said equations. .

The following empirical methods were used:

Document Analysis: Academic records, previous evaluations and materials related to logarithmic and exponential functions were reviewed. This allowed us to obtain information about the students' previous performance on these topics.

Observation: During the teaching unit classes, observations were made to evaluate how students interacted with the concepts of logarithmic and exponential functions. Special attention was paid to their participation, understanding and application of the contents.

Interviews: Students were carried out to inquire about their perceptions, difficulties and learning strategies in relation to logarithmic equations.

RESULTS

The analysis of the main theoretical references about the influence that differentiated attention exerts on the process of solving logarithmic equations with emphasis on preventing the difficulties that students have in terms of the preceding content with a high impact on the solution process. of these equations, allowed the determination of the elements of mathematical knowledge from previous grades and didactic units previous to the content related to logarithmic equations in 11th grade, with a high impact on the solution process of this type of equations and which then are related:

-Process of solving linear, quadratic, fractional, trigonometric, and irrational equations.

-Properties of powers, properties of radicals, monotony of logarithmation, properties of logarithms.

-Definition of logarithm of a real number, base and argument of a logarithm, solution set of an equation, basic domain of an equation, equivalent transformations, equivalent equations.

The educational diagnosis, for its part, provided a comprehensive view of the students' level of preparation in logarithmic and exponential functions, which helped guide pedagogical strategies and design specific interventions to improve their understanding and mastery of this important mathematical content.

Procedure for promoting the learning of logarithmic equations in pre-university education

In each of the classes, independent study exercises were guided individually with each content diagnosed with difficulties, which made it possible to work preventively on these contents and thus prevent them from manifesting themselves negatively in the solution of the logarithmic equations. In the first class related to logarithmic equations, they were informed about the practical application of the following content:

· In 1935, the American seismologist Charles Richter developed a logarithmic scale that bears his name and is widely used to calculate the magnitude of the intensity of an earthquake using the equation:

$$M = \frac{2}{3} \log \frac{E}{E_0}$$

where E is the energy released by the earthquake, measured in joules, and E_0 is the energy released by a very mild

earthquake that has been standardized as:

$$E_0 = 10^{4.40} \text{ joules}$$

· Rocket theory of flight is used in advanced mathematics and physics to show that the speed v of a rocket upon shutdown (when fuel is exhausted) is given by

$$v = c \ln \frac{W_i}{W_b}$$

where c is the engine exhaust velocity, W_i is the starting weight (fuel, structure and payload), and W_b is the consumed weight (structure and payload)

· The concentration of the hydrogen ion of a substance is related to its acidity and basicity. Because hydrogen ion concentrations vary over a very wide range, a compressed *pH scale is used*, which is defined:

$$pH = -\log [H^+]$$

where $[H^+]$ is the concentration of the hydrogen ion, in moles per liter.

As can be noted, each of the phenomena addressed was solved using common and natural logarithms.

Subsequently, they were given the procedure to follow to solve logarithmic equations in their two analytical variants.

Variant 1:

1- Obtain all logarithms with the same base

2- Transpose all logarithms for one member and the independent terms for the other.

3- Obtain only one logarithm by applying properties.

4- Apply definition of logarithm

5- Solve the equation that originates

6- Perform the check

7- Express the solution.

To understand this methodological procedure, it was proposed to analyze the following example

Solve the following equation

$$\log(x-2) = \log(9x-18) + 1$$

As can be seen, all the logarithms involved have the same base. All logarithms are transposed to the left side of the equation obtaining:

$$\log(x-2) - \log(9x-18) = 1$$

$$\log\left(\frac{x-2}{9x-18}\right) = 1$$

$$10^1 = \frac{x-2}{9x-18}$$

The following transformations can be performed on the obtained equation:

$$10(9x-18) = x-2$$

$$90x-180 = x-2$$

$$89x = 178$$

$$x = 2$$

So the possible solution to the equation is $x=2$

Checking, we obtain that $x=2$ does not belong to the domain of definition of the initial equation, so the possible solution does not constitute a solution of the equation, therefore $S = \emptyset$.

Subsequently, variant 2 was proposed to the students to solve a logarithmic equation.

Variante 2 Reducción de la ecuación a la forma $\log_a T(x) = \log_a M(x)$:

1. Obtener solo un logaritmo en cada miembro de la ecuación aplicando propiedades, con la característica que tienen la misma base.
2. Igualar los argumentos.
3. Resolver la ecuación que surge.
4. Realizar la comprobación.
5. Expresar la solución.

Una vez se propuso la segunda variante, se le exoneró.

Resolver la siguiente ecuación

$$\begin{aligned}\log(x-2) + \log(x-3) &= 1 - \log 5 \\ \log(x-2) + \log(x-3) &= \log 10 - \log 5 \\ \log(x-2)(x-3) &= \log \frac{10}{5}\end{aligned}$$

Al igualar los argumentos se obtiene la siguiente ecuación

$$\begin{aligned}(x-2)(x-3) &= \frac{10}{5} \\ (x-2)(x-3) &= 2 \\ x^2 - 5x + 6 &= 2 \\ x^2 - 5x + 4 &= 0 \\ (x-4)(x-1) &= 0 \\ x-4 = 0, x-1 &= 0\end{aligned}$$

De donde se tiene que $x=4$, $x=1$, son las posibles soluciones de la ecuación dada. Se realizó la verificación.

La posible solución $x=1$ no es una solución de la ecuación ya que no pertenece al dominio de definición de la ecuación.

Lo contrario sucede con $x=4$, que es una solución ya que pertenece al dominio de definición de la ecuación.

Para la investigación, se utilizaron diferentes procedimientos utilizados por autores que han estudiado la resolución de ecuaciones logarítmicas en la educación pre-universitaria, las diferentes variantes para su solución y las acciones a seguir por el docente en cada una de estas variantes que fueron tenidas en cuenta.

Con el grupo de 80 estudiantes del IPU "Federico Engels" de Pinar del Río se implementó el procedimiento durante el año escolar 2023. Se realizó un análisis de los resultados obtenidos en el examen final de la asignatura, en el contenido relacionado con las ecuaciones logarítmicas, en términos del promedio de la asignatura y los resultados se compararon con los de otros estudiantes del año que no fueron sujetos de la investigación, así como con los resultados de otros años.

Adicionalmente, se realizó una comparación entre los resultados obtenidos en el examen de preparación realizado por el Departamento Provincial de Educación, para contrastar el nivel alcanzado para enfrentar el examen de ingreso a la universidad, con los obtenidos en este tipo de verificación de los cursos (2020 a 2023).

Finalmente, se estableció una comparación entre los resultados obtenidos en el examen final y el examen de preparación en el año académico 2022-2023 en relación con el examen de ingreso del año en el que se evaluó este contenido, para corroborar la influencia del procedimiento con un énfasis en la prevención de dificultades con las ecuaciones logarítmicas.

Los resultados obtenidos respecto al promedio de la asignatura basado en 20 puntos en el examen evaluado en el examen final, relacionado con las ecuaciones logarítmicas durante tres cursos consecutivos, fueron los siguientes:

Table 1- Comparison of the results corresponding to the average grade (2019-2022)

	2019-2020 academic year	2020-2021 academic year	2021-2022 academic year
Average grade	14.5	13.8	13.5

The average grade based on 20 points in this topic of the 80 students in the 2022-2023 academic year was 18.7, higher than that obtained in the previous ones (Table 1.) while the rest of the students in the 11th grade of the IPU "Federico Engels" from Pinar del Río who were not the subject of this research obtained an average grade of 13.3 points, very similar to the behavior of previous courses.

Table 2 shows the results achieved in the preparation exams for the entrance exam to higher education in the Pinar del Río province during the 2019-2020, 2020-2021, 2021-2022 academic years, and the results obtained in the 2022-2023 academic year where the research was developed, in the objective related to logarithmic equations.

Table 2- Comparison of the results corresponding to the average grade (2019-2022)

	2019-2020	2020-2021	2021-2022	2022-2023
Average grade	13.3	14.1	13.7	18.6

Finally, Table 3 shows the comparison regarding the average grade between the final exam, a preparation exam for the entrance exam proposed by the provincial directorate of education, in the 2022-2023 academic year and the average grade achieved in the exam. of entry in the year 2020 in the content related to logarithmic equations.

Table 3- Comparison of the results corresponding to the exams (final, preparation and entry)

	Final exam 2022-2023	Preparation exam 2022-2023	Entrance exam year 2020
Average grade	18.7	18.6	12.1

DISCUSSION

In general, the results allowed us to confirm that, if the teaching-learning process of logarithmic equations in 11th grade is developed with emphasis on preventing difficulties in future performance, so that a more efficient differentiated attention process is promoted, qualitatively higher levels of learning are achieved.

All of the above allows us to understand that an important element to promote motivation for learning these contents at this level of education is the orientation of investigative work and practical work, exhibitions of historical aspects and the development of Mathematics, knowledge contests. , the use of mathematical assistants, the holding of conferences and discussions with teachers and specialists, the analysis of the historical results of these contents in previous courses in the subject exams and entrance exams to higher education, among other activities. These elements promote motivation for mathematics and its learning, which is in line with González (2022) for whom motivation regulates the direction, the goal object and the intensity or activation of behavior, which is why it is considered by some authors that Contents whose meaning is not asserted do not become motivation for learning.

The existing correlation between attention to individual differences with emphasis on preventing students' difficulties in future performance related to the solution of

logarithmic equations in the 11th grade of the IPU "Federico Engels" of Pinar was also corroborated. del Río with success in learning and motivation to study the subject, a result proposed by Klingber (1972).

This is why attention to individual differences based on work in the group must be taken into account, as well as preventive work to enhance the process of differentiated attention and elevate student learning to qualitatively higher levels. Additionally, what is described by Contreras et al. (2021) who propose, among other aspects, that different situations that may appear in any area of life enable the student to argue and identify fundamental ideas, establish judgments, as well as analyze the validity of arguments about certain propositions, achieving greater learning of the students, had its correspondence in the results obtained, evidenced in the need for the performance of mathematical tasks to require an adequate distribution of mental processing and memory resources, as well as the use of ordered and hierarchical strategies, which imply a progressive fit of some procedures in others.

It is also important to consider that, to achieve better learning, the way in which the contents are taught plays an essential role, since they are assimilated more quickly if they are carried out taking into account individual differences, taking preventive work and taking into account the variety of ways in which these can be presented. In this direction, it is necessary for the teacher to promote the autonomy and management of knowledge on the part of the student, considering as the center of the learning process, active and dynamic entities, protagonists in their learning process in a way that allows them to encourage critical thinking to from the development of the argumentation and promotion of different strategies, as proposed by Guzmán et al. (2022)

On this basis, students are presented with the content related to logarithmic equations and the elements of knowledge that underlie the teaching of this content, such as the

types of equations studied in previous units and courses (linear, quadratic, fractional, irrational, trigonometric), properties of logarithms, monotonicity of the logarithmic function and the definition of the logarithm of a real number greater than zero.

The structuring of the content taking into account individual differences in correspondence with the development achieved by each student, constitutes one of the regulations of the current class that has the purpose of raising the quality of the educational process with emphasis on the instructive, as well as the training of values and attitudes based on the preventive work developed with each of them, so it is suggested to include in each class subsystem, tasks that respond to each of the levels of development achieved by the students.

It is considered, then, that the teaching-learning process of Mathematics in pre-university fosters the development of each student's thinking, based on the influence in the emotional sphere, making it possible for the contents learned to have meaning and meaning. personal, so that they serve as a basis for the new content that happens and that will be necessary for your actions in life. This criterion corresponds to what was proposed by Munayco and Solís (2021) for whom the understanding, invention and resolution of problems allow the progressive development of mathematical skills; There is a relationship of interdependence between them.

It is possible for the student to understand the exercise and begin the solution process using the guidance given by the teacher. The teacher proposes the necessary questions, creating a favorable socio-psychological climate to ensure that the student is involved in the formulation context and can find the most rational strategies that allow you to give the answer according to the initial conditions posed in the exercise so that you can assess whether the path found during the solution process is correct.

In general, it is concluded that, in the teaching-learning process of logarithmic equations, the student constitutes an active entity playing a leading role, where he reveals to the teacher the level of achievements achieved in learning these contents, and the limits of their learning, so that it allows the teacher to develop the profile of their learning and update the diagnosis of each of them with respect to the content treated.

The results obtained suggest that those obtained in the upcoming higher education entrance exams, related to this topic, be investigated at the time they are proposed and compare them with those historically obtained both in the 11th grade, and with the exams. of preparation.

The differentiated attention in the teaching-learning process of logarithmic equations in 11th grade with emphasis on preventive work constitutes a vitally important element in the learning and motivation for the study of these contents at this level of teaching, shown through the average grade of the students. The proposed procedure reveals the importance of its application in achieving qualitatively higher levels in mathematics learning.

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Authors' contribution:

The author participated in the process of reviewing the state of the art of research, writing, preparing the original draft, writing and revising the final version of the article.

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