**Gamow-Teller $\beta^+ \text{ decay properties of } A=98 \text{ isobars near } ^{100}\text{Sn doubly magic core}**

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**Abstract**

In this work, we have realized some spectroscopic calculations in the framework of the nuclear shell model, in order to estimate the Gamow-Teller (GT) $\beta^+$ decay of $A=98$ proton rich isobars in $^{100}\text{Sn}$ mass region near rp-process path. The calculations are carried out by means of Oxbash nuclear structure code, taking into account the monopole effect in the studied mass region. The obtained results are then compared to the available experimental data.

**Key words:** nuclear structure; strontium 100; monopoles; o codes; gamow-teller rules; beta-plus decay.

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**Introduction**

Nuclei in doubly magic regions near drip lines have been subject of both theoretical and experimental studies that aim to well understand the spectroscopic behaviour of nuclear forces in such regions. Therefore, the heaviest $N=Z$ magic core $^{100}\text{Sn}$ situated in the proton drip line and near the rp-process provides important information on nuclear structure and astrophysics [1, 2].

The first excited states of $^{98}\text{Cd}$ were first identified by Gorska et al. (1997). They have proposed its experimental level scheme and associated $J^\pi=8^+$ to the $T_{1/2}=0.48$ µs isomer [3]. In 2004, Blazhev et al. [4] have observed a core excited $12^+$ isomer and measured $T_{1/2}=0.23$ µs and 0.17 µs in $^{98}\text{Cd}$. Huyse et al. (1978) discovered the $^{98}\text{Cd} b^+$ daughter with the LISOL facility, following the irradiation of $^{92}\text{Mo}$ with a 110 MeV $^{14}\text{N}$ [5]. The $^{98}\text{Ag} b^+$ descendent was identified and reported by AtenJr and Vries-Hamerling (1955) [6].

Brown and Rykaczewski (1994) theoretically studied GT $b^+$ decay properties of nuclei near $^{100}\text{Sn}$ mass region[1]. They have presented their spectroscopic calculations using SNB interaction [1] in fpg space model, for odd-even and odd-odd nuclei in near $^{100}\text{Sn}$. Covelo et al. (2006) have performed shell model calculations for nuclei in the vicinity of $^{100}\text{Sn}$ core using an interaction derived from CD-Bonn one in gdsh space model [7]. They obtained good agreement with the experimental data.

In this paper, we have studied the $b^+$ Gamow-Teller decay properties of $A=98$ proton rich isobars near the doubly magic tin-100 core by means of shell model calculations using Oxbash nuclear structure code [8].

**Theoretical framework**

The shell evolution is the result of the interactions between the magic core, and the adding nucleons [9, 10, 11] or the so-called monopole effect described by Poves and Zuker [12] and defined in terms of the two-body interaction. Hence, the consideration of this effect can reproduce the missing nuclear properties of nuclei far from stability. They proposed to express the monopole Hamiltonian of the system in terms $\hat{H}$:

$\hat{H}_m = \sum_{i,j} V_{ij} \hat{c}_i \hat{c}_j$, 

where $V_{ij}$ is the monopole interaction between nucleons $i$ and $j$. The effect of the monopole interaction is to shift the levels close to the magic numbers, leading to the formation of magic nuclei.
\[
H_{\text{iso}} = \sum_{i} n_{i} e_{i} + \sum_{i \in \tau} (a_{i} n_{i} + b_{i} T_{i})
\]

\[
V_{\text{iso}}^{\text{iso}} = \frac{\sum (2J+1) V_{j} (j,j')}{\sum (2J+1)}
\]

sand/or \( t \) denote a proton and/or a neutron orbit. \( n_{i} \) and \( T_{i} \) refers, respectively, to the number and the isospin operator defined by A. P. Zuker (2003) [9, 14] as a function of the monopole Hamiltonian diagonal part \( V_{\text{iso}}^{\text{iso}} \) [11], and \( \tau \) \((\tau')\) stands for proton or neutron.

In this work, we have used the recent single particle energies (SPEs), and considered the mass and the monopole effects to introduce some modifications on the two body matrix elements (TBMEs) of the original interaction \( jj45 apn \) from \( ^{78}\text{Ni} \) mass region (Jensen [16, 17]). These TBMEs are used in order to calculate the monopole terms:

\[
V_{12,34}^{\text{iso}} \approx -430 \text{ keV}, \quad V_{12,34}^{\text{pp}} \approx 112 \text{ keV} \quad \text{and} \quad V_{24,34}^{\text{iso}} \approx -18 \text{ keV}
\]

to modify TBMEs chosen basing on the energetic sequence of the single particle space. Using the resulting interaction \( jj45 \text{pm} \) and the original one, some calculations are carried out in order to reproduce the nuclear and \( \beta^{+} \) Gamow-Teller transition properties of \( A=98 \) isobars (Figure 1).

The \( \beta^{+} \) decay rate, \( \lambda_{\beta^{+}} \), of transition from the state \( i \) to the state \( j \), and the allowed \((f\ell)_{\beta^{+}}\) values can be estimated using [18]:

\[
\lambda_{\beta^{+}} = \frac{\ln 2}{\langle f \rangle_{\beta^{+}}} f_{\beta^{+}} \quad \text{and} \quad \frac{1}{\langle f \rangle_{\beta^{+}}} = \frac{1}{(\langle f \rangle_{\beta^{+}})^{\text{GT}}} + \frac{1}{(\langle f \rangle_{\beta^{+}})^{\text{F}}}
\]

\( f_{\beta^{+}} \) denotes the \( \beta^{+} \) decay phase space factor. \((\langle f \rangle_{\beta^{+}})^{\text{GT,F}}\) are the \((\langle f \rangle_{\beta^{+}}) \) values for Gamow-Teller (GT) and Fermi (F) transitions, which can be expressed in terms of the matrix elements \( M_{\text{GT}} \) and \( M_{\text{F}} \) used to estimate the GT and F transition probabilities [19].

\[
B_{\text{GT}} = \frac{g_{\pi}^{2}}{2J_{i} + 1} |M_{\text{GT}}|^{2}, \quad B_{\text{F}} = \frac{g_{\pi}^{2}}{2J_{i} + 1} |M_{\text{F}}|^{2} \quad \text{with} \quad g_{\pi} = \frac{1}{1.26}
\]

Results and discussion

In this work, we have performed shell model calculations, using the new interaction \( jj45 \text{pm} \) in \( \pi(0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2}), \quad \nu(0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2} \text{and} 1h_{11/2}) \) N-50 model space using \( ^{100}\text{Sn} \) as a magic core. The experimental single hole and single particle energies taken, respectively, from \( ^{99}\text{In} \) for protons and \( ^{101}\text{Sn} \) for neutrons are used as a starting point to calculate the effective single particle energies [20, 21] using in the interaction.

The calculated configuration changes between the initial and final states indicate that the important values are observed for \( 9g_{9/2} \) and \( 9g_{7/2} \). Which means that the \( ^{96}\text{Cd} \) and \( ^{96}\text{Ag} \) protons in the \( 9g_{9/2} \) populate the \( ^{98}\text{Ag} \) and \( ^{98}\text{Cd} \) neutrons in \( 9g_{7/2} \) respectively (Figure 2).
Most of the strength of the $^{98}_{\text{Cd}} \rightarrow ^{98}_{\text{Ag}}$ GT transition, limited by a $Q_{\text{EC}}$ value of 5.43 MeV, is located in two peaks concentrated at about 1.5 MeV and 4.5 MeV. For the $^{98}_{\text{Ag}} \rightarrow ^{98}_{\text{Pd}}$ GT transition, limited by a $Q_{\text{EC}}$ value of 8.25 MeV, it is located in two peaks concentrated at about 2.5 MeV and 4.5 MeV (Table 1).

Table 1. Experimental and calculated $T_{1/2}$ for $^{98}_{\text{Cd}}$ and $^{98}_{\text{Ag}}$

<table>
<thead>
<tr>
<th></th>
<th>$T_{1/2}$ Exp(s)</th>
<th>$T_{1/2}$ Cal(s)</th>
<th>$Q_{\text{EC}}$ (MeV)</th>
<th>$B(GT)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{98}_{\text{Cd}}$</td>
<td>9.2</td>
<td>98.920</td>
<td>4.410</td>
<td>14,566</td>
</tr>
<tr>
<td>$^{98}_{\text{Ag}}$</td>
<td>47.5</td>
<td>2.61</td>
<td>7.23</td>
<td>1,337</td>
</tr>
</tbody>
</table>

Conclusion

This study is based on the energetic spectra and Gamow-Teller properties calculations, for odd-odd $A=98$ isobars, with few hole protons and neutrons in their valence spaces. The calculations are realized in the framework of the nuclear shell model, by means of Oxbash nuclear structure code. Using the jj45apn original interaction of the code, we carried out some modifications based on the proton-neutron monopole interaction to get jj45m interaction. The calculated energetic spectra are in agreement with the experimental data for $^{98}_{\text{Cd}}$ and $^{98}_{\text{Pd}}$; however, the spin and parity of $^{98}_{\text{Ag}}$ ground state are not reproduced. The $^{98}_{\text{Cd}}$ and $^{98}_{\text{Ag}}$ protons in the $\pi g_{9/2}$ populate the $^{98}_{\text{Ag}}$ and $^{98}_{\text{Pd}}$ in $\nu g_{7/2}$, respectively.

The obtained half lives of the studied transitions have the magnitude of the experimental ones. The studied GT transitions are limited by $Q_{\text{EC}}$ values of 5.43 MeV and 8.25 MeV. Most of the strength of the $^{98}_{\text{Cd}} \rightarrow ^{98}_{\text{Ag}}$ GT transition is located in two peaks concentrated at about 1.5 MeV and 4.5 MeV. Most of the strength of the $^{98}_{\text{Ag}} \rightarrow ^{98}_{\text{Pd}}$ GT transition is located in two peaks concentrated at about 2.5 MeV and 4.5 MeV.

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References


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