\( \beta^+ \) decay properties of A=100 isobars

Fatima Benrachi, Nadjet Laouet
Laboratoire Physique Mathématique et Subatomique, Université Frères Mentouri Constantine 1 (Algeria).
fatima.benrachi@umc.edu.dz

Abstract
The estimation of spectroscopic properties of neutron-deficient nuclei in the A=100 tin mass region is needed for the understanding of the rp-process path and the experimental exploration of the nuclear landscape. In order to evaluate some spectroscopic properties of the Gamow-Teller \( \beta^+ \) decay of neutron deficient isobars of A=100, we have performed shell model calculations by means of Oxbash nuclear structure code. The jj45pn valence space used consists of nine proton and neutron orbitals. The calculations included few valence hole-proton and particle-neutron \( \pi g_{9/2} \) and \( v g_{7/2} \) orbitals respectively, in \( ^{100}\text{Sn} \) doubly magic core. Effective interaction deduced from CD-Bonn one is introduced taking into account the nuclear monopole effect in this mass region. The results are then compared with the available experimental data.

Key words: nuclear structure; strontium 100; monopoles; 0 codes; beta-plus decay; neutron-deficient isotopes

Propiedades de la desintegración \( \beta^+ \) de isóbaros con A = 100

Resumen
La estimación de las propiedades espectroscópicas de los núcleos deficientes en neutrones en la región de masas de estaño A = 100 es necesaria para la comprensión del camino del proceso rp y la exploración experimental de la estructura interna de los núcleos. Con el fin de evaluar algunas propiedades espectroscópicas de la desintegración \( \beta^+ \) de Gamow-Teller en los isótopos de estaño deficientes en neutrones con A = 100, hemos realizado cálculos del modelo de capas mediante el código de estructura nuclear de Oxbash. El espacio de valencia jj45pn utilizado consiste en nueve orbitales de protones y neutrones. Los cálculos incluyeron pocos hueco-protón y partícula-neutrón de valencia en orbitales \( \pi g_{9/2} \) y \( v g_{7/2} \) respectivamente, en un núcleo \( ^{100}\text{Sn} \) doblemente mágico. La interacción efectiva deducida de CD-Bonn se introduce teniendo en cuenta el efecto monopolar nuclear en esta región de masa. Los resultados se comparan luego con los datos experimentales disponibles.

Palabras clave: estructura nuclear; estroncio 100; monopoles; códigos 0; desintegración beta positiva; isótopos deficientes en neutrones.

Introduction
The study of the nuclei with few hole protons and particle neutrons near the heaviest nucleus \( ^{100}\text{Sn} \) (N=Z) was the main of several theoretical and experimental works that aimed to give a global description of nuclear structure. These isobars are some of the best candidates for giving us opportunity to develop our understanding of nuclear structure due to the proximity of the magic numbers, the proton drip-line and the end of the rp-process. One of the most interesting aspects of the proton-rich nuclei is show in some of the unusual features of their Gamow-Teller beta decays. Very scarce experimental information in this region are available [1]. The nuclear shape changes rapidly in this region and gives rise to long-lived isomers with small excitation energies [2, 3]. With the development of advanced experimental tools such as isotope-separation on-line (ISOL) and in-beam spectroscopy with large \( \gamma \)-arrays including ancillary particle detectors and by exploiting spallation and heavy ion induced fusion-evaporation reactions, many of the nuclei along the Z, N = 48–50 isotonic and isotopic chains were be produced with relatively high rates [2, 4, 5]. Investigations of testing effective interaction, exploring the evolution of the shell structure and beta decays have been performed in some works [3, 6-10].

The present analysis deals with three neutron-deficient nuclei \( ^{100}\text{In}, ^{100}\text{Cd} \) and \( ^{100}\text{Ag} \) neighbours of \( ^{100}\text{Sn} \) core. Shell model calculations are been performed in the full model space that include the proton and neutron orbits below and above the closed shells Z= N = 50...
respectively. This model space enables one to reproduce the shape of the GT strength distribution. In these nuclei, protonspartly fill the \( g_{9/2} \) orbit and the spin-flip transformation \( 2g_{9/2} \rightarrow g_{9/2} \) allowed by Pauli principle forms the Gamow-Teller (GT+) states in the daughter nucleus. The main part of the strength turns out to lie within the QEC window, thus becoming accessible to the \( \beta^+ \) decay [10].

The shell model calculations of the hole-particle multiplets in these nuclei were performed with code OXBASH [11] supplementing available experimental data.

### Theoretical framework

Nowadays, studies of radioactive decays are an integral part of investigations atomic nucleus in order to better understand the physical phenomena governing its behavior. The \( \beta^+ \) decay process plays a major role in determining fundamental quantities such as the period, the mass or the energy of the excited levels helping to understand nuclear interaction and the characterization of populated states. Beta decay is a weak interaction process mediated by the well understood \( \tau \) and \( \sigma \) operators that govern the observed Fermi (F) and Gamow-Teller (GT) transitions, respectively. Under some experimental conditions, Charge Exchange (CE) reactions proceed by F and GT transitions, although mediated by the strong interaction. Consequently, \( \beta^+ \) decay and CE processes can be very similar. The main advantage is that \( \beta^+ \) decay provides absolute \( B(GT) \) strength values, whilst the CE reactions extend our knowledge to excitation energy regions above the Q\( _a \) value. Also, CE reactions can only be studied readily on stable target nuclei, whereas by definition \( \beta^+ \) decay is from unstable nuclei. Beta decays of nuclei “southeast” of \( ^{100}\text{Sn} \) are characterized by large decay energies (QEC) and pure Gamow-Teller (GT) transitions transforming a \( g_{9/2} \) proton to a \( g_{7/2} \) neutron. These features make studies of these decays especially important to test the nuclear shell model in general and its predictions of the GT strength in particular. The experimentally derived quantity which can be directly compared to the theoretical predictions, is the GT strength function. Gamow-Teller beta-decay strengths provide relatively clear information about the structure of nuclear wave functions because the associated operators are simple and selective, coupling only a few single-nucleon orbits to each other. In addition, these operators do not couple to the first-order admixtures of “excluded” configurations into the “active” space [12].

The theoretically predicted GT strength is defined as the squared matrix element of the free \( \sigma \) operator acting between the wave functions of the initial \( |i> \) and the final \( |f> \) states:

\[
B(GT) = \frac{g_\alpha^2}{2j_1+1} |\langle f|GT|i\rangle|^2
\]  

\[
\langle f|GT|i\rangle = \sum_n \delta_f^{\pm} n
\]

The factor \( g_\alpha = 1.2\lambda \) is the axial-vector coupling constant of the weak interaction and \( \lambda \) is the angular momentum of the initial state.

The quantity \( t_{1/2} \) value of an allowed beta-decay transition is related to Gamow-Teller force \( B(GT) \) (1), \( f \) is a phase-space integral that contains the lepton kinematics [13] and \( t_{1/2} \) is the half-life.

The focus on the evolution of shell structure in nuclei was increased, in order to understand the appearance of new magic numbers. The effect of the addition of nucleons on the single particle states can lead to spectroscopic properties nuclei that the realistic interactions derived from the N-N force fail to reproduce. The interactions between the supposed inert core and the adding nucleons were be solved by the consideration of the monopole effect introduced by Poves and Zuker [14]. Those were assumed to need drastic revisions of the realistic two-body potentials and proposed to separate the Hamiltonian of the system into two parts:

\[
H = H_m + H_m^t
\]  

where \( H_m \) denote the monopole part of the Hamiltonian and \( H_m^t \) the monopole one. is expressed in term of the average energies over the configurations of s and t orbits with \( T = 1 \) for proton–proton and neutron–neutron, and \( T = 0 \); 1 for proton–neutron parts, see [15–16] for more details.

\[
\nu^T_{\alpha} = \sum (2j+1)\langle\langle j,j'|\langle j',j|\nu_T|j',j\rangle\rangle\nu^-\nu^T\delta_{\alpha}\nu^T
\]

The two body matrix element \( \langle j,j'|\nu_T|j',j\rangle \) arisen from the interaction between the particles in the orbits s and t. It can be extracted from the proton and/or the neutron separating energies of neighbouring nuclei [17]. The \( \nu^T_{\alpha}(3) \) defined the diagonal 2b part of the monopole Hamiltonian associated to a function of the average two body matrix elements (TBMEs).

The main aim of this paper is to present some calculations on nuclear properties of \( ^{100}\text{In}, ^{100}\text{Cd} \) and \( ^{100}\text{Ag} \) isobars, focusing attention on the levels schemes and the \( \beta^+ \) decay properties. This study isrealized in the framework of the nuclear shell model by means of Oxbash nuclear structure code [11].

### Spectroscopic calculations and discussion

We have used the \( ^{100}\text{Sn} \) core and the full model space where proton holes are allowed to occupy the \( \pi(0f_{5/2}, 1p_{3/2}, 1p_{1/2} \) and \( 0g_{9/2} \) ) \( z,28 \) orbitals and the neutron particles occupy the \( v(0g_{9/2}, 1d_{5/2}, 1d_{3/2} ,2s_{1/2} \) and \( 1h_{11/2} \) ) \( n,50 \) orbitals. The experimental single particle energy (pSHE and nSPE) values are taken from \( ^{99}\text{In} \) for protons and \( ^{100}\text{Sn} \) for neutrons [18], respectively.
Considered the dependence mass factor \((78/100)^{0.3}\), the two body matrix elements (TBMEs) of the original interaction \(jj45apn\) from \(^{78}\)Ni mass region \([19,20]\) are scaled. The resulting TBMEs are used in order to calculate the monopole terms. Therefore

\[
V_{1p_3/2d_{5/2}} = -430 \text{ keV} \quad \text{and} \quad V_{1p_3/2d_{5/2}} = -110 \text{ keV} \quad \text{and} \quad V_{1p_3/2d_{5/2}} = -20 \text{ keV}
\]

are used to modify

\[
\tau^J_\nu \left( \frac{\lambda}{\lambda'} \right) \quad \text{and} \quad \tau^J_\nu \left( \frac{\lambda}{\lambda'} \right) \quad \text{and} \quad \tau^J_\nu \left( \frac{\lambda}{\lambda'} \right)
\]

TBMEs, respectively. These TBMEs are chosen basing on the energetic sequence of the single particle space. Using the new interaction \(jj45m\), some calculations are carried out in order to reproduce the nuclear properties of the three \(A=100\) isobars cited above. The obtained levels schemes are showing on figure 1. The \(jj45m\) interaction reproduce the sequence of levels for even-even \(^{100}\)Cd nucleus. A good agreement between calculated energies and experimental ones are observed. This interaction cannot reproduce the spin of the experimental ground state for \(^{100}\)In\((6^+)\) and \(^{100}\)Ag\((5^+)\) isobars, it gives \(5^+\) and \(2^+\) for this state, respectively. Also, it can’t reproduce the sequence of levels for \(^{100}\)Ag nucleus.

Figure 2 shows that theoretical occupancy changes (with \(\Delta J = 1\)) of valence nucleons in the states of parent and daughter nuclei are maximum of proton hole-1\(g_{9/2}\) and neutron particle-1\(g_{7/2}\) orbitals respectively. A small contribution of the proton orbitals 2\(p_{3/2}\) and 2\(p_{1/2}\) is obtained in the case of \(^{100}\)Cd decay. The GT decay towards \(^{100}\)Cd will populate the two quasi particle (2qp) configuration \(2p_{1/2}^2 + 1g_{7/2}^2\).

**Table 1.** \(T_{1/2}\) \(\beta^+\) decay calculated of \(^{100}\)In and \(^{100}\)Cd nuclei and the experimental ones

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>(T_{1/2}) (s) Exp</th>
<th>(T_{1/2}) (s) Cal</th>
<th>(Q\beta^+) (MeV)</th>
<th>(\Sigma B(GT))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{100})In</td>
<td>5.8 ± 0.2</td>
<td>0.57</td>
<td>8.86</td>
<td>8.64061</td>
</tr>
<tr>
<td>(^{100})Cd</td>
<td>49.1 ± 0.5</td>
<td>2.61</td>
<td>2.921</td>
<td>9.9277</td>
</tr>
</tbody>
</table>

Figure 3 shows that theoretical occupancy changes (with \(\Delta J = 1\)) of valence nucleons in the states of parent and daughter nuclei are maximum of proton hole-1\(g_{9/2}\) and neutron particle-1\(g_{7/2}\) orbitals respectively. A small contribution of the proton orbitals 2\(p_{3/2}\) and 2\(p_{1/2}\) is obtained in the case of \(^{100}\)Cd decay. The GT decay towards \(^{100}\)Cd will populate the two quasi particle (2qp) configuration \(2p_{1/2}^2 + 1g_{7/2}^2\).

**Summary**

This study is based on the energetic spectra and Gamow-Teller \(\beta^+\) decay properties calculations for nuclei near \(^{100}\)Sn, with few hole protons and particle neutrons in their valence spaces. The calculations are realized in the framework of the nuclear shell model, by means of Oxbash nuclear structure code. Using the \(jj45apn\) original interaction of the code, we carried out some modifications based on the monopole effect to get \(jj45m\) new interaction. This interaction reproduce the energe-
tic spectrum for $^{100}$Cd nucleus, it can’t reproduce the ground state for odd-odd $^{100}$In and $^{100}$Ag isobars and the sequence of levels for $^{100}$Ag nucleus. The complete distributions of B(GT) strengths of excitation states in daughter nuclei are located in centered peaks and limited by $Q_E.C.$ values. Their values are practically the same in both decays and different from experimental value for $^{100}$In. The calculated $T_{1/2}$ values obtained with the standard quenching factor of 0.77 for $\pi$ operator related to the $\beta^+$ decay of $^{100}$In and $^{100}$Cd are far from the experiment ones by a factor ten and depend substantially on the model used.

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References


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