Development of a semi-empirical method to determine the efficiency of a gamma radiation detector for point sources

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Abstract
In the present work, we introduce the theoretical development and a new approach to validate a novel method to calibrate the efficiency of a gamma radiation detector for point sources, which we have named the "Efficiency Extrapolation Method".
The method consists in the determination of the detector efficiency using a set of monoenergetic gamma sources, which we will refer to as reference efficiencies. From these values, we will extrapolate the detector efficiency to the complete energy range using the first principle physics of gamma radiation detection theory. Therefore the proposed method corresponds to a semi-empirical one.
The determination of reference efficiencies must be done experimentally, but in this work, simulations were performed using FLUKA code. The reference energies will be: 59,54 keV, 661,66 keV and 1 274,54 keV associated to the isotopes ^{241}Am, ^{137}Cs and ^{22}Na respectively. The second part of the method, the extrapolation from the reference energies to the gamma range, will be done over the energies emitted by ^{152}Eu and ^{133}Ba.
In general, the results are very good. While the obtained results for energies 53,16 keV and greater than 344,3 keV show an excellent agreement, the results obtained for energies in the middle range are only good.

Key words: *point sources; gamma radiation; high-purity GE detectors; calibration; accuracy.*

Desarrollo de un método semi-empírico para determinar la eficiencia de un detector de radiación gamma para fuentes puntuales

Resumen
En el presente trabajo presentamos el desarrollo teórico y un nuevo enfoque para validar un nuevo método para calibrar la eficiencia de un detector de radiación gamma para fuentes puntuales, al cual hemos llamado el método de extrapolación de la eficiencia.
El método consiste en la determinación de la eficiencia del detector usando un conjunto de fuentes gamma monoenergéticas, a las cuales llamaremos eficiencias de referencia. Desde estos valores, extrapolaremos la eficiencia del detector al rango completo de energías usando la física de primeros principios de la teoría de detección de radiación gamma. Por lo tanto el método propuesto corresponde a un método semiempírico.
La determinación de las eficiencias de referencia debe hacerse experimentalmente, pero en este trabajo, se realizaron simulaciones usando el código FLUKA. Las energías de referencia serán: 59,54 keV, 661,65 keV y 1 274,54 keV asociadas a los isótopos ^{241}Am, ^{137}Cs y ^{22}Na respectivamente.
La segunda parte del método, la extrapolación desde las energías de referencia hacia el rango gamma de las energías, será realizada sobre las energías emitidas por los isótopos ^{152}Eu y ^{133}Ba.
En general, los resultados son muy buenos. Mientras que los resultados obtenidos para las energías de 53,16 keV y mayores que 344,3 keV muestran un excelente acuerdo, los resultados obtenidos para las energías del rango intermedio son solo buenas.

Palabras clave: *fuentes puntuales; radiación gamma; detectores de GE ultrapuro;*

Introduction

Gamma spectrometry using a Hiper Pure Germanium (HPGe) detector is the most popular technique for the determination and quantification of the radioactive nuclei present in radioactive samples. When we want to know the quantities of the radioactive nuclei present in a point-like source, we need to have knowledge of the full energy peak (FEP) efficiency of the detector for a point for the different energies emitted by the radioactive sample under study. The position of both sources must be the same.

We are developing a new semi-empirical method to determine the FEP efficiency by means of extrapolation from a set of experimental points of the detector efficiency. The experimental points are used to discover the average traveled path by the photons in the detector. Then, using the linear attenuation coefficient of the detector material for the interesting energy, we determined the FEP efficiency for such energy.

For the testing, we have used FLUKA code [1] to determine the efficiencies of the detector and XCOM database [2] to know the attenuation coefficients.

In a first work [3] we submitted a study for the application and validation of the method using one reference energy. Consequently, in that work the agreement between the obtained values by the proposed method and the expected values was worse while the energies were farther from the reference energy. In this second work, we will apply the method using three references energies in different domains along the gamma energy range with the aim of decreasing such differences.

Semi-empirical method to determine the efficiency of a gamma detector:

The total intrinsic efficiency of a detector is defined as

$$\epsilon_{int}(E,\vec{r}) = 1 - \exp(-\lambda(E).d(\vec{r})) \quad , (1)$$

where: E is the energy of the detected photons, *r* is the source position, λ is the total linear attenuation coefficient for the energy E and d the path traveled by photons in the detector.

If we know the total intrinsic efficiency, $\epsilon_{int}(E_1,\vec{r})$ and the total linear attenuation coefficient for an energy E_1, $\lambda(E_1)$, the average path length can be determined by

$$\overline{d}(\vec{r}) = \frac{\ln(1-\epsilon_{int}(E_1))}{-\lambda(E_1)} \quad , (2)$$

Now if we know the linear attenuation coefficient for another energy, $\lambda(E_2)$, we can determine the total intrinsic efficiency of the energy E_2 by the following expression

$$\epsilon_{int}(E_2,\vec{r}) = 1 - \exp(-\lambda(E_2).d(\vec{r})) \quad , (3)$$

where the average traveled is known, because it was already determined experimentally in Equation (2).

Finally, we can determine the peak intrinsic efficiency of the detector using the relation between total efficiency and peak efficiency given by,

$$\epsilon_{int,\,peak} = \frac{P}{T}.\epsilon_{int,\,total} \quad ,$$

where *(P/T)* is the peak to total ratio.

The peak to total ratio must be determined by simulation as Moens indicates in his work [4]. As our work is developed using simulations, this last part of the method is not performed yet.

Materials and methods

Simulated setup

Using the FLUKA code we simulated a bare cylindrical germanium detector and the isotropic monoenergetic radioactive sources. The dimensions of the simulated detector are 3,84 cm height and 2,375 cm radii.

Determination of the reference efficiencies and the reference average traveled paths

First, we determined the total absolute efficiencies by statistical count for three reference energies well distributed all along the interest gamma energy range by simulation using FLUKA code [1]. These energies are 59,54 keV, 661,65 keV and 1 274 keV corresponding to the emissions of ^{241}Am, ^{137}Cs and ^{22}Na respectively. Then, considering the solid angle subtended for the source over the detector the total intrinsic efficiencies were determined for such energies. Finally, using total attenuation coefficient obtained from XCOM database [2], we determined the reference average traveled path of photons into the detector.

The average traveled path must be the same for all energies, but this can suffer variations as we can observe in table 1. For this reason, the results obtained in the first work were worse for farther energies from the reference energy, because we only used one energy reference (661,65 keV), and as we can see the average

Table 1. Calculations results.

Radioactive Source	Reference energy (keV)	Distance source-detector (cm)	Reference total intrinsic efficiency	Total attenuation coefficient λ (cm^{-1})	Average traveled path *d* (cm)
^{241}Am	59,54	30	9,77 E-01	1,03 E+01	3,66 E-01
^{137}Cs	661,6	30	6,89 E-01	3,70 E-01	3,15 E+00
^{22}Na	1 274	30	5,74 E-01	2,67 E-01	3,20 E+00

traveled path changes in table 1. In the table, we show the reference energies used for the extrapolation of the efficiency, together with the efficiency obtained in the simulation and the total attenuation coefficient. In the last column, we show the average traveled path of the photon into the detector.

For example, for the high energies range the multiple interactions into the detector increases the average traveled path. On the opposite case, for low energies, the average traveled path decreases due to the fact that photons rarely cross the detector, and cross a small path when these impact in the edge or the detector.

This can clearly be seen in the results presented in the next section in table 2. We have maintained the sign of relative bias to show such effect. The reason for the positive sign for all efficiencies obtained by the method for energies lesser than the reference energy is because these are greater than the expected value of the efficiency obtained by statistical count. Therefore the average traveled path of such photons must be less than the obtained from the reference efficiency. The opposite happens for energies greater than the reference energy. This is deduced from the change of sign when passing the reference energy.

Table 2. Comparison between the total intrinsic efficiencies obtained by statistical count and by extrapolation using the proposed method.

Radioactive Source	Photon Energy (keV)	Relative difference % Reference energies (keV)		
		^{241}Am 59,54	^{137}Cs 661,65	^{22}Na 1 274
^{133}Ba	53,16	1,50	2,07	2,07
	79,61	-15,09	3,26	3,26
	80,99	-16,51	3,28	3,28
^{152}Eu	121,8	-49,87	5,61	5,64
	244,7	-74,50	3,64	4,05
	344,3	-77,90	2,78	3,33
	444,0	-79,66	1,44	2,05
	778,9	-82,23	-0,29	0,46
	867,4	-82,60	-0,38	0,39
	964,0	-82,99	-0,77	0,02
	1 086	-83,31	-0,61	0,22
	1 112	-83,39	-0,66	0,17
	1 212	-83,67	-0,97	-0,13
	1 299	-83,86	-1,01	-0,15
	1 408	-84,12	-1,43	-0,56

Determining the best energy range for the extrapolation from different reference energies

In this table, we can see that the best result for e 53,16 keV energy is achieved when using the 59,54 keV reference energy. The best result for energies within the 121,8 - 867,4 keV range is achieved when using 661,65 keV reference energy, and for energies greater than 867.4 the optimal extrapolation is when using 1 274 keV as reference energy.

Results

In table 3 here below, we show the results for the total intrinsic efficiency obtained from this method. In the mentioned table, the domains in which the extrapolation was carried out are indicated by horizontal lines.

Table 3. Comparison between the intrinsic efficiencies by the semi-empirical method and by the relative method.

Reference Energy (keV)	Energy (keV)	Source-detector distance (cm)		
		10	20	30
59,54	53,16	1,33	1,65	1,50
661,6	79,61	4,67	3,73	3,26
	80,99	4,90	3,78	3,28
	121,8	9,44	6,39	5,61
	244,7	5,81	4,50	3,64
	344,3	3,63	2,89	2,78
	444,0	1,70	1,76	1,44
	778,9	-0,77	-0,42	-0,29
	867,4	-1,68	-0,20	-0,38
1 274	964,0	0,97	0,41	0,02
	1 086	0,74	0,49	0,22
	1 112	0,48	0,02	0,17
	1 212	0,03	0,21	-0,13
	1 299	-0,19	0,14	-0,15
	1 408	0,01	-0,12	-0,56

We can observe that the results are better for the farthest source-detector distance, specifically, the best results are for the 30 cm source-detector distance. Regarding the results along the complete energy range of photons and the same source-detector distance, the results obtained for 53,59 keV and for the energies ranging from 444,0 keV to 1 408 keV are excellent. While the obtained results for energies ranging from 79,61 keV to 344,63 keV are only good, in the case of 30 cm source-detector distance, and acceptable, in the case of 10 cm source-detector distance. The worst result is obtained for 121,8 keV, which achieves 9,44% of relative bias for 10 cm source-detector distance, even so, this result is not completely bad.

Discussion

The proposed method, named as Efficiency Extrapolation Method, shows an excellent agreement for the majority of energies, except for a little range in the low energy section, where the results are acceptable. However, the results could be improved using a pair of reference energies among the energy range.

Finally, the application of the method using three reference energies has considerably improved the results regarding the first work, in which only one reference energy was used.

Conclusions:

We have determined the total intrinsic efficiency of a simulated detector using the Efficiency Extrapolation Method.

We have applied the efficiency extrapolation method using three reference energies, each one for the low, medium and high energy range. And, as we expected, the bad results obtained in the first work [3] were improved using such set of reference energies.

The agreement between the total intrinsic efficiencies obtained by the method and the obtained by the statistical count is excellent for energies greater than 661,65 keV and for 53,16 keV, while for energies between these the results are good, however, these can be improved. Therefore, we have tested successfully the efficiency extrapolation method.

More reference energies should be used in the low energy range for the extrapolation over energies lesser than 444,0 keV for improving the agreement in this region.

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