Revista Ciencias Técnicas Agropecuarias, ISSN -1010-2760, RNPS-0111, Vol. 22, No. 3 (julio-agosto-septiembre, pp. 12-17), 2013

ARTÍCULO ORIGINAL

Formulación de un modelo friccionante-cohesivo de suelo por el método de elementos discretos

Formulation of a frictional-cohesive soil model by the Discrete Element method

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ABSTRACT. Friction and cohesion in efforts that take place during soil deformation calculations. For interactions are considered normal force, tangential cohesive friction. The maximum shear is governed by the criterion of Mohr-Coulomb failure. Model calibration is performed by obtaining the relationship between micro and macroscopic parameters used in the model. The calibration of the friction was carried out by simulating the direct shear test of a sample of soil micro different friction coefficients. To calibrate cohesion used different values of cohesion at the micro and the relationship was obtained by modifying the micro friction values by simulating the biaxial compression test. The friction results showed a linear relationship regarding the variation of the micro value. Instead cohesion was linearly affected by the change in the quadratic micro-cohesion and friction over micro.

Keywords: Tillage, Modeling, Simulation, Discrete Element Method, Draft force.

RESUMEN. En el presente trabajo se propone un modelo de elementos discretos que incluye el efecto de la fricción y la cohesión en los esfuerzos que tienen lugar durante la deformación del suelo. Para los cálculos se consideran las interacciones entre la fuerza normal, tangencial, cohesiva y friccionaste. El valor máximo de fuerza cortante es regulado mediante el criterio de falla de Mohr-Coulomb.La calibración del modelo se realizó mediante la obtención de la relación existente entre los parámetros micro y macroscópicos usados en el modelo. La calibración de la fricción se realizó por medio de la simulación del ensayo de corte directo de una muestra de suelo a diferentes valores de fricción micro. Para calibrar la cohesión se utilizaron diferentes valores de cohesión a escala micro y se obtuvo la relación existente al modificar los valores de fricción micro mediante la simulación del ensayo de compresión biaxial. Los resultados de la fricción mostraron una relación lineal respecto a la variación de su valor micro. En cambio la cohesión estuvo afectada de forma lineal por la variación en la micro-cohesión y cuadrática respecto a la fricción micro.

Palabras clave: simulación, labranza, modelos, elementos discretos, fuerza de tiro.

INTRODUCTION

Soil in a discontinuous model is commonly represented by assemble of spherical particles with bonds that can break up and reform under external loading. Developed by Cundall, (1971), the method was developed for simulated the rock slope stability and soil structures. When suitable contact force models are used and microscopic model parameters are carefully selected, the DEM procedure can offer a realistic description of soil mechanical behavior. However, as a real soil is composed of a huge number of small particles with complex shapes, it is inevitable to represent such a granular material as an assembly of larger idealized particles. Micro-macro parameter relationships resolve the difference in particles sizes and shapes, but introduce more complexity into the method (Asaf *et al*., 2007).

Simulations of soil mechanical behaviour generally showed good qualitative agreement with experimental tests, but accurate quantitative results were not yet obtained (Shmulevich, 2010). The differences were related to the differences between the size and shape of the soil particles with those in the model.

Considering the characteristics of the clay soil obtained by Lopez *et al*., (2012), especially its considerable cohesion, the present study is focused on the virtual tests to calibrate the model.To obtain the relationship between soil micro-macro friction and cohesion the simulation of direct shear and biaxial compression test were performed in the standard configuration.

METHODS

Model formulation

The soil-tillage tool interaction dynamics were implemented in the *DEMeter++* Software (Tijskens *et al.*, 2003). This program provides tools for building and computing the interaction between simple and complex bodies, including contact detections, relative bodies' velocities, contact force calculation, time integration, etc. Written in C++ this software has the possibility to utilise open source libraries, compile own applications and create executable files. The output of the calculations can be saved according to the kind of application providing easy and comfortable management of the entire system. The Mohr-Coulomb criterion of shear failure is used to regulate the force in tangential direction:

$$F_t^{max} < F\_n \cdot tan(\phi_\mu) + c_\mu \tag{1}$$

where:

F_t^{ma} = tangential force limit [N],
F_n= normal force [N],
ϕ_μ = micro-friction angle [°],
c_μ = micro-cohesion [N].

The variation in normal force DF_n at the contact point has elastic and a viscous component:

$$\Delta F_n = k_n \cdot u_n + \eta_n(u_n/\Delta t) \tag{2}$$

where:

k_n= normal stiffness [N/m],
u_n = normal displacement [mm],
η_ν = viscous damping coefficient [kg/s],
Δt = time step [s].

The stiffness parameter is calculated by the model proposed by Liao (1997) based on the stress-strain relationship. To obtainthe values of the dimensionless parameters (α_k=2.65, β_k= 0.65, γ_k=1.0) in the Liao model, a set of compression tests was carried out by Hentz *et al.*(2004). The model was also verified in a mathematical study by Feng *et al.* (2007) with the purpose of validating the DEM prediction capacity. From this modelthe stiffness in normal direction was determined as:

$$k_n = \frac{E_{ab} \cdot A_{int}}{D_{eq}} \left[\frac{1 + \alpha_k}{\beta_k(1+\nu) + \gamma_k(1-\alpha_k)} \right] \tag{3}$$

where:

E_{ab}= equivalent Young's modulus of the materials in contact [Pa];
A_{int}= interaction surface [m^2]:
D_{eq}= equivalent distance between the two particles [m];
ν = Poison's ratio;
α_k,= loading path;
β_k, = softening factor;
γ_k=interaction range.

Similar to the normal force, the variation of tangential force ΔF_s was calculated considering the elastic and plastic effects:

$$\Delta F_s = k_s \cdot \Delta u_s + \eta_s(\Delta u_s/\Delta t) \tag{4}$$

where:

k_s= tangential stiffness [N/m];
Δu_s= increment of tangential overlapping [m];
η_s= viscous damping coefficient in tangential direction [kg/s].

The magnitude of the tangential stiffness also depends on the value of the normal one Hentz *et al.*, (2004) and is calculated by the following equation:

$$k_s = k_n \left(\frac{1 - \alpha_k \cdot \nu}{1 + \nu} \right) \tag{5}$$

The viscous damping is obtained as:

$$\eta_i = \beta_i \cdot 2 \cdot \sqrt{\frac{m_a \cdot m_b}{m_a + m_b}} \tag{6}$$

where:

i = right subscript representing normal and tangential direction;
β = coefficient of viscous damping;
m_a, m_b = mass of the objects in contact [kg].

Finally, the equations to calculate the model parameters of micro-friction ϕ_μ and micro-cohesion c_μ are obtained by modeling the soil macro-behaviour during the simulation of biaxial compression test and direct shear test.

Geometrical model of virtual shear test

In order to reproduce the direct shear test with a DEM model a virtual reproduction of the direct shear test apparatus (Figure 1) was made. A set of samples for the simulation were also generated by spherical particles to test the micro-friction ϕ_μ effect over macro samples strength.The geometrical setup for the simulation (Figure 1a) was divided in two main sections: upper and lower box; each one with a cylindrical cavity at the centre.

The soil specimen is obtained by sequential steps, starting with spheres generated inside the cylindrical cavity of the shear apparatus following a hexagonal compacted spatial distribution. The centre of each particle is obtained by assigning an independent position in the (*x, y, z*) axes for an initial number of 4,000 particles. Dimensions of the particles were calculated by random distribution of the radii between 1.0 to 1.5 mm. In the next step the particles falling under the action of the gravity force.

The particles number is fixed at 3,680 to obtain the desire height of the sample. The generated soil sample hasa cylindrical shape of 70 mm diameter by 60 mm height. From this point, the geometrical model is ready for simulation of direct shear test. The model macro-parameters (Table 1) were calculated by means of the regression equations defined by Lopez *et al.*(2012) at the soil intermediate physical condition according to soil moisture w = 21.3% and dry bulk density ρ_d= 1.18 g/cm^3.

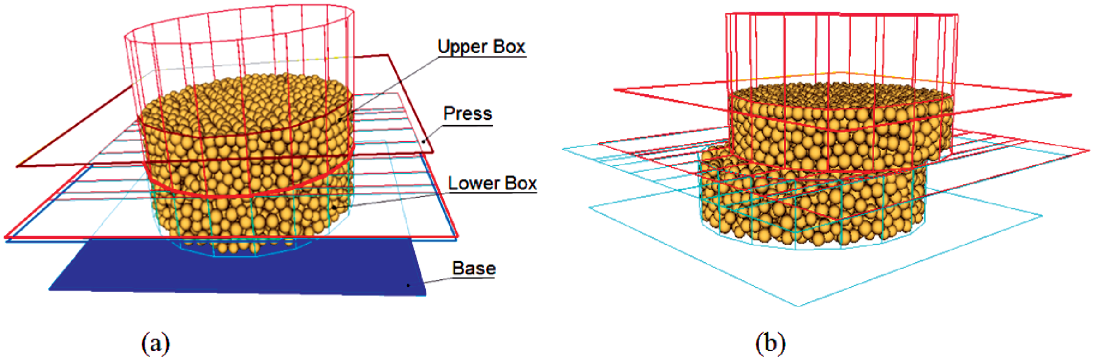

FIGURE 1. Virtual direct shear test. Geometric model (a), sample cutting simulation (b).

TABLE 1. Model macro-parameters for soil at intermediate condition

Parameters	Values	Units
t_{50}	539.9	kPa
E	53.9	MPa
ν	0.28	
c	72.6	kPa
c_a	5.42	kPa
ϕ	22.2	°
δ	15.3	°

During the simulation the sample is cutting in the transversal direction as shown in Figure 1b.The shear operation takes place moving the upper box over a length of 20 mm at aconstant velocityof 1.3 mm/s. The press surface ensures that shear take places at different normal load.

The relationship between micro and macro friction was obtained by changing the value of the micro-friction angle in several tests computed with the same soil parameters defined on the Table 1. Accordingly, four values of ϕ_μ were tested (2, 4.2, 6.4 and 8.6°), each one at a normal pressure of 30, 50, 70 and 90 kPa resulting in a total of sixteen simulation tests.

Geometrical model of biaxial compression test

Soil by nature is a frictional and cohesive material; its strength is a result of both contributions. This behaviour is introduced in the model by the micro-friction and micro-cohesion parameters acting at the particle level. By means of the simulation of biaxial compression test the relationship between real soil cohesion and micro-cohesion for the model is obtained. In order to find the variation of soil cohesion the simulation is carried out in unconfined conditions, which means without lateral pressure. With the compression-displacement curve resulting from the test, the cohesion calibration procedure is made following the Mohr-Coulomb criterion of failure.

The procedure to form the virtual samples was the same as the one used for direct shear test, i.e. coordinates generation, particles falling, compression, and decompression. An auxiliary cylinder of 50 mm diameter is used as a mould (Figure 2a) for a virtual sample made by 5,500 spherical particles. Before running the compression test the cylinder (mould) is removed from the installation and a total of 5,350 particles form the final sample after the height is fixed. The particles followa random radius distribution between 1.5 to 2 mm, sized at 50 mm diameter by 100 mm height.

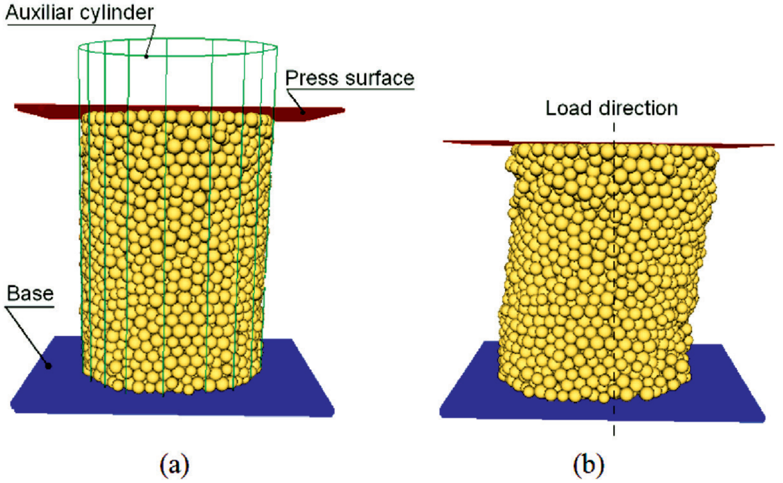

FIGURE 2. Compression test setup. Cylindrical specimen (a), deformation under load (b).

Finally, the geometrical model of the biaxial compression test is composed by the press surface, the base, and the soil sample (Figure 2a). In the experiment the press surface is moved at a constant velocity of 1 mm/min in a downward direction (Figure 2b). The reaction forces are computed until the sample reaches a deformation of 20% in vertical direction.

The micro-macro relationship for the cohesion parameter is obtained by testing the compression virtual model at three different values of micro-friction c_μ = 40, 80 and 120 kPa;

To analyse the effect of particle friction over micro-cohesion the biaxial compression test was executed four times changing only the micro-friction angle at ϕ_μ = (2, 4.2, 6.4 and 8.6°) and keeping constant the inter-particle cohesion.

RESULTS AND DISCUSSION

Macro and micro-friction relationship

The shear strength at failure is selected as the maximum tension reached during the test. After this point the sample strength decreases.When the normal pressure increases, a new value of shear strength is attained; the results for each combination are plotted in the shear-normal plane as shown in Figure 3.

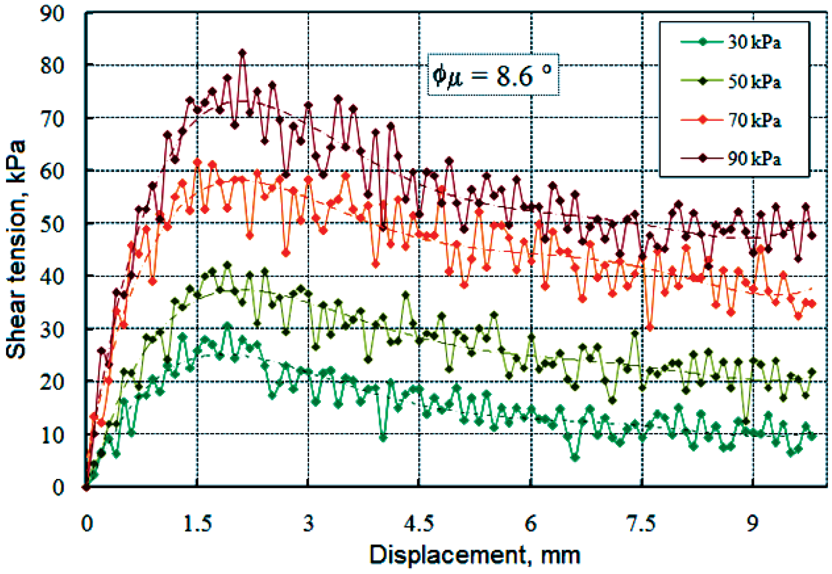

FIGURE 3. Force-displacement curves at of simulated shear test at different values of normal pressure.

Shear strength at failure increases linearly with respect to normal pressure (Figure 4), in agreement with the laboratory experiments. In addition, an increment in the micro-friction parameter enhances the values of shear tension.

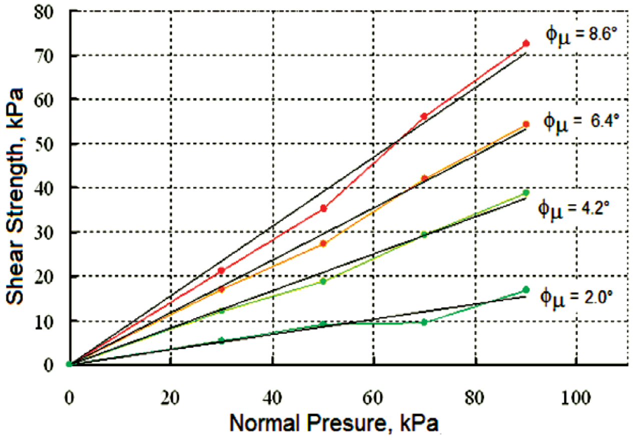

FIGURE 4. Friction angle envelopes from simulated test using different micro-friction angles.

The slope of the regression line of shear strengths gives the internal friction angle of the material corresponding with the micro-friction tested in the model. The relationship between macro and micro friction is obtained fromthe regression line describing by the soil internal friction angle versus the micro-friction parameter (Figure 5).

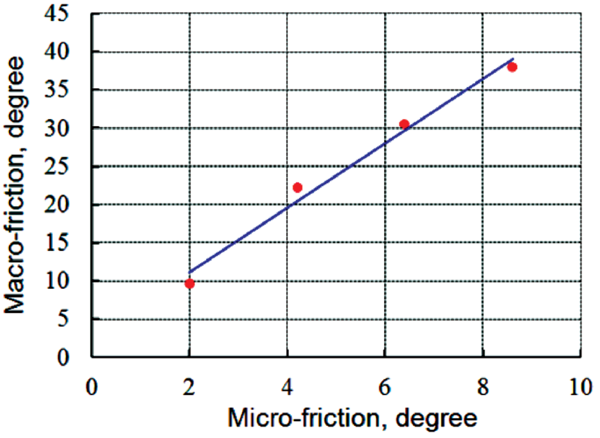

FIGURE 5. Macro-friction as a function of micro-friction angle from the model.

The linear regression equation characterises the particular condition tested, influenced mainly by the particles size distribution and sample porosity. The equation to define the relationship between internal frictionϕ and micro-frictionϕ_μ is written as:

$$\phi_\mu = 0.23\phi - 0.53 \qquad (7)$$

By this equation the values of inter-particles friction ϕ_μ in the DEM model can be calculated as a function of the real soil internal friction angle ϕ. The model should be able to reproduce the soil shear strength according to the normal pressure applied.

Macro and micro cohesion relationship

Shear strength at failure is localized in the compression-displacement curve as the maximum value achieved during the test. Particle accommodation takes place at the start of the test without substantial increases in forces (Figure 6). After the first millimetre of deformation the sample strength increases until reaches the maximum value. After this point the tension gradually decreases. These results are in agree with those obtained in clay soils by experimental mechanical tests (Herrera *et al.*, 2008; Gonzalez, 2011).

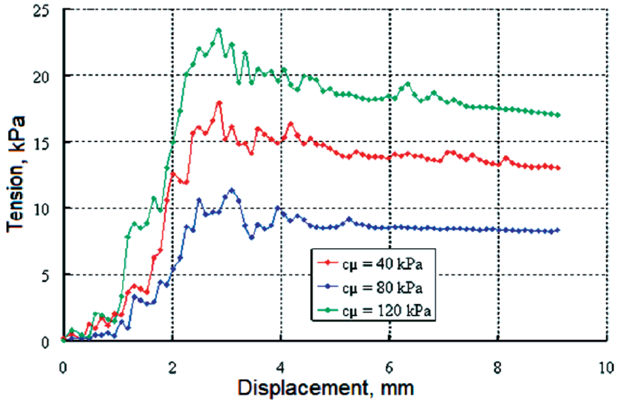

FIGURE 6. Pressure-displacement curves for different values of micro-cohesion.

The relationship between micro and macro cohesion for different incremental values of internal friction is shown in Figure 7.

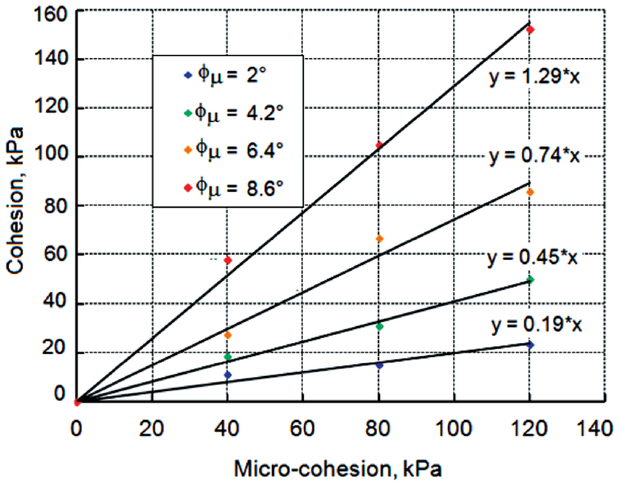

FIGURE 7. Specimens' cohesion obtained from incremental values of micro-cohesion.

Increasing soil micro-cohesion results in a linear increment of soil cohesion determined by the slope in the regression equation obtained for each value of micro-friction under test. In order to find an equation to involve both parameters of soil strength, the slope of c-c_μ versus the corresponding values of micro-friction are plotted.

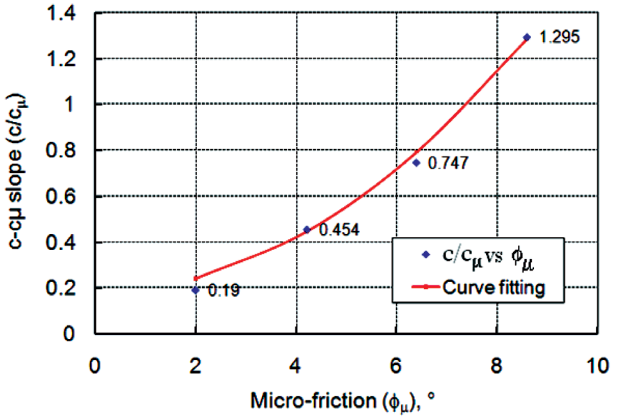

FIGURE 8. Regression curve for c-c_m as a function of the micro-friction parameter.

As shown in Figure 8, non-linear behaviour distinguishes this relationship. As the slope of c-c_μ increases, the micro-friction angle increases too. The regression equation is written as:

$$\frac{c}{c_\mu} = a\,\phi_\mu^{\ 2} + b \qquad (8)$$

where:

ϕ_μ = micro-friction [°]:

a = 0.15;

b = 0.18[ ].

As shown in the equation 8, the magnitude of macro-cohesion resulting from the model increases proportionally with the increment of inter-particles cohesion and friction angle, a quadratic relationship was also found to respect of micro-friction angle ϕ_μ. This results agree with those obtained by Delenne (2004) in a study of the influence of the friction parameter in a cohesive bonds.

CONCLUSIONS

- To calculate the mechanical response of a cohesive soil at particle level in the DEM model the system forcescan be formed by normal force, shear force, friction force, cohesion force and gravity force. Shear force at failure is defined by the Mohr-Coulomb criterion.
- Using the geometrical elements from DEMeter++ software a virtual reproduction of the direct shear test and compression was made. The strength obtained in the virtual soil samples during the simulation are used to find the relationship between micro and macro parameters.
- The shear strength at failure during the simulation of shear test was found to increases linearly with respect to the normal pressure. The same behaviour is shown as the micro-friction parameter increase. A linear regression equation characterises the relationship between micro-friction and macro-friction,influenced by factors associated to particles size distribution and sample porosity.
- Alinear increment in soil macro-cohesion was the result of

increases micro-cohesion parameter in the model. However a quadratic behaviour was found as the micro-friction increase.

- The statistical equations obtained through the simulation allow calculating the micro values for the DEM model started from the macro-parameters of the soil valid for soil models with the same configuration of particle arrangement and force system.

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Recibido: 10 de septiembre 2012.
Aprobado: 14 de junio de 2013.

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